

## MTH 165 COLLEGE ALGEBRA

Creating open expressions and using those expressions to write equations involving one or two variables to solve problems and applications will be integrated throughout this course. Examples from other disciplines will be incorporated whenever possible. A comprehensive departmental final exam testing the degree of mastery of the following course objectives is required.

### 1. Equations and Inequalities

- 1.1 Review solving formulas for a single variable.
- 1.2 Review solving rational equations in one variable.
- 1.3 Review the concept of complex numbers.
- 1.4 Review writing complex numbers in  $a+bi$  form, and review adding, subtracting, multiplying, and dividing complex numbers.
- 1.5 Solve quadratic equations in one variable.
  - a. Review solving by factoring.
  - b. Review solving by the square root method.
  - c. Review solving by completing the square on  $ax^2 + bx + c = 0$ , where  $a = 1$ .
  - d. Solve by completing the square on  $ax^2 + bx + c = 0$ , where  $a \neq 1$ .
  - e. Review solving by the quadratic formula.
  - f. Review using the discriminant to classify the roots.
- 1.6 Solve factorable polynomial equations in one variable.
- 1.7 Solve radical equations in one variable.
  - a. Review solving radical equations with one radical term.
  - b. Solve radical equations with at most two radical terms.
- 1.8 Solve equations in one variable involving rational exponents.
- 1.9 Solve equations in one variable that are quadratic in form.
- 1.10 Review solving linear absolute value equations in one variable.
- 1.11 Review solving compound linear inequalities in one variable, expressing the solution using set-builder notation, interval notation, and on the real number line.
- 1.12 Review solving linear absolute value inequalities in one variable, expressing the solution using set-builder notation, interval notation, and on the real number line.

### 2. Functions and Graphs

- 2.1 Determine algebraically and graphically whether a given relation is a function.
- 2.2 Review the use of function notation.
- 2.3 Given a formula for a function, evaluate the function at a specified value of the independent variable, and when a given expression is input for the independent variable.
- 2.4 Graph each of the following basic functions:  
 $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = x^3$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sqrt[3]{x}$ , and  $f(x) = |x|$ .
- 2.5 Given a function represented by a formula and/or a graph, determine the zeros, the x-intercepts, and the y-intercept.
- 2.6 Find the domain of a given function algebraically.
- 2.7 Find the domain and range of a function from its graph.
- 2.8 Determine algebraically and graphically whether the graph of a given equation is symmetric with respect to the x-axis, y-axis, and/or origin.

- 2.9 Evaluate a piecewise function for a specified value of the independent variable.
- 2.10 Graph piecewise functions with constant, linear, and/or quadratic pieces.
- 2.11 State the piecewise definition of the absolute value function.
- 2.12 Review writing the equation of a line using slope-intercept form and point-slope form.
- 2.13 Use translations to sketch a graph which is a horizontal and/or vertical shift of the graph of any of the basic functions listed in objective 2.4, and then determine the domain and range of the translated function.
- 2.14 Find the sum, difference, product, and quotient of two functions, and determine the domain of each.
- 2.15 Find the composition of two functions.
- 2.16 Determine graphically whether a given function is one-to-one.
- 2.17 Given a formula for a one-to-one function, determine the formula for its inverse function.
- 2.18 Given the graph of a one-to-one function, sketch the graph of its inverse function.
- 2.19 Given the equation of a circle, determine the center and radius (using completing the square if necessary), and then sketch the graph of the circle.

### 3. Polynomial Functions

- 3.1 Given a quadratic function in the form  $f(x) = ax^2 + bx + c$ , review finding the vertex, the axis of symmetry, and the intercepts, and use this information to graph the function. Use the vertex or completing the square to put the function in the form  $f(x) = a(x-h)^2 + k$ .
- 3.2 Given a quadratic function in the form  $f(x) = a(x-h)^2 + k$ , find the vertex, the axis of symmetry, and the intercepts, and use this information to graph the function.
- 3.3 Solve applied word problems involving quadratic functions.
- 3.4 Use the Leading Coefficient Test to determine the end behavior of a polynomial function.
- 3.5 Find the zeros of a factorable polynomial function.
- 3.6 Review long division of polynomials.
- 3.7 Use synthetic division to determine the quotient and remainder when a polynomial function  $p(x)$  is divided by  $x - c$  where  $c$  is a positive or negative rational number, and then determine whether or not  $x - c$  is a factor of  $p(x)$ , and whether or not  $c$  is a zero of  $p(x)$ .
- 3.8 Given a polynomial function  $p(x)$ , use the Remainder Theorem to determine the value of  $p(c)$ .
- 3.9 Given a polynomial function  $p(x)$ , apply the Factor Theorem to deduce that  $c$  is a zero of  $p(x)$  if and only if  $x - c$  is a factor of  $p(x)$ .
- 3.10 Given a polynomial function of degree at most 4 and one of its zeros, apply the Factor Theorem and synthetic division to find the remaining zeros.
- 3.11 Given a polynomial function of degree at most 4 with integer coefficients, use the Rational Zeros Theorem to list all possible rational zeros of the function, and then use synthetic division to determine the actual zeros of the function.
- 3.12 Sketch the graph of a polynomial function of degree at most 4 by determining and using its end behavior, x-intercepts, and y-intercept.
- 3.13 Given that  $p(x)$  is a polynomial function with *real* coefficients and  $a + bi$  is a complex zero of  $p(x)$ , deduce that  $a - bi$  is also a zero of  $p(x)$ .

- 3.14 Use the Fundamental Theorem of Algebra to find a polynomial function of lowest degree with real coefficients that has given zeros.
- 3.15 Solve factorable quadratic inequalities and inequalities involving rational expressions, expressing the solution using set-builder notation, interval notation, and on the real number line.
- 3.16 Given a verbal description that specified variables are related by direct, inverse, and/or joint variation, write the appropriate equation.
- 3.17 Solve applied word problems involving direct, inverse, and/or joint variation.

#### **4. Exponential and Logarithmic Functions**

- 4.1 Graph an exponential function of the form  $f(x) = b^x$ , where  $b > 0, b \neq 1$ , and determine the function's domain, range, y-intercept, and horizontal asymptote.
- 4.2 Graph the functions  $f(x) = e^x$  and  $f(x) = 10^x$ .
- 4.3 Solve applied word problems using the formula for interest compounded  $n$  times per year and the formula for interest compounded continuously.
- 4.4 Express logarithmic equations in exponential form.
- 4.5 Express exponential equations in logarithmic form.
- 4.6 Graph a logarithmic function of the form  $f(x) = \log_b x$ , where  $b > 0, b \neq 1$ , and determine the function's domain, range, x-intercept, and vertical asymptote.
- 4.7 Graph the functions  $f(x) = \ln x$  and  $f(x) = \log x$ .
- 4.8 Use translations to sketch a graph which is a horizontal and/or vertical shift of the graph of either an exponential or a logarithmic function, and then determine the domain, range, and asymptote of the translated function.
- 4.9 Explain that  $f(x) = b^x$  and  $g(x) = \log_b x$  are inverse functions, and that their graphs are reflections of each other over the line  $y = x$ .
- 4.10 Apply the inverse rules:  $b^{\log_b x} = x$  (for  $x > 0$ ) and  $\log_b(b^x) = x$ .
- 4.11 Given an expression involving a single logarithm, use the Product Rule, the Quotient Rule, and/or the Power Rule to expand the expression, assuming all variables are positive.
- 4.12 Given an expanded expression involving logarithms, use the Product Rule, the Quotient Rule, and/or the Power Rule to condense the expression into a single logarithm, assuming all variables are positive.
- 4.13 Approximate common and natural logarithms using a calculator.
- 4.14 Solve exponential equations.
- 4.15 Solve logarithmic equations.
- 4.16 Solve applied word problems involving exponential growth and decay, including examples where given information must be used to solve for the value of the growth/decay rate  $k$ .

#### **5. Systems of Equations**

- 5.1 Solve linear-quadratic systems of equations in two variables algebraically and graphically.
- 5.2 Solve systems of two quadratic equations in two variables algebraically and graphically.