MTH 210 CALCULUS I – Course Objectives

Students will be expected to demonstrate an understanding of Calculus beyond the manipulation of symbols, apply Calculus to practical problems and use current technology throughout the course. They will demonstrate their understanding of Calculus using four approaches--geometric, numerical, algebraic and verbal. A comprehensive departmental final exam testing the degree of mastery of the following course objectives is required.

1. General Objectives

1.1 Recognize and manipulate functions given in numerical, graphical, and analytical forms.

1.2 Give reasonable approximations for values of functions, their limits, derivatives and integrals and be able to discuss the error involved.

1.3 Use graphing calculator technology to explore the behavior of functions, limits, derivatives and integrals; to find numerical approximations for limits, derivatives and integrals; and to aid in solving problems and verifying solutions.

1.4 Express Calculus concepts, and explain and interpret results in well-written sentences.

1.5 Interpret the derivative as the limit of a difference quotient that gives the slope of a linear approximation to a graph at a point, and as instantaneous rate of change.

1.6 Interpret a definite integral both as a limit of a Riemann Sum that is the difference in areas above and below the x-axis, and as the net amount of change in the function whose rate of change is the integrand.

1.7 Explain the relationship between the derivative and the definite integral as it is expressed in both parts of the Fundamental Theorem of Calculus.

1.8 Use derivatives and integrals to model and solve applied problems.

1.9 Use the sign, magnitude, and units of measurement of a solution to an applied problem to assess its reasonableness.

2. Review of functions and graphs with applications

2.1 Review the definitions of function, domain, and range, and relate them to functions given in tabular, graphical, or algebraic form.

2.2 Recognize and apply the following functions given any one of a table of values, a graph, or a formula for the function: sine, cosine, tangent, linear, quadratic, exponential, logarithm

2.3 Use the above functions in significant applications.

2.4 Compose and evaluate functions involving arithmetic combinations of sine, cosine, tangent, linear, quadratic, exponential, and logarithmic functions using the correct order of operations.

2.5 Graph any combination of sine, cosine, tangent, linear, quadratic, exponential, and logarithmic functions choosing appropriate settings for the view window.

3. Limits and Continuity

3.1 Write an intuitive, English definition of limit., e.g., Def: $\lim_{x \to a} f(x) = L$

"*f(x) can be made arbitrarily close to L by making x sufficiently close to, but not necessarily equal to, a.*" or "*As x approaches a from both sides, the value of f(x) approaches L.*"

3.2 Recognize that a function is continuous at c for functions given numerically, graphically, and algebraically

3.2.1 Numerically: "Close values of the domain lead to close values of the range."

3.2.2. Graphically: "a continuous function is one whose graph can be drawn without lifting pencil from paper."

3.2.3 Algebraically: $\lim_{x \to c} f(x) = f(c)$

3.3 Approximate limits, including left and right hand limits at c and limits at plus and minus infinity, numerically and graphically, using a calculator.

3.4 Using rules, evaluate limits algebraically, including left and right hand limits at c and limits at plus and minus infinity.

3.5 Describe a function that is unbounded at c in terms of infinite limits.
3.6 Describe vertical asymptotes using infinite limits.
3.7 Describe horizontal asymptotes using limits at infinity.

4. Derivatives

4.1 Explain average rate of change in general and in a variety of contexts, including average velocity. Describe average rate of change as a difference quotient of output and input values and as the slope of a secant line.
4.2 Compute average rate of change from a table of values, a graph, or a formula.
4.3 Describe instantaneous rate of change as the limit of average rate of change and estimate instantaneous rate of change from a table, from a graph, and from a formula using a calculator.
4.4 Express an instantaneous rate of change as a derivative and find a variety of instantaneous rates of change, including velocity, speed, and acceleration, by evaluating derivatives.
4.5 Write the limit definition of the derivative of a function and find the derivative of a polynomial function using the definition.
4.6 Know derivatives of powers of x, sine and cosine, exponential and logarithmic functions.
4.7 Apply Sum, Difference, Constant Multiple, Product, Quotient and Chain Rules to find derivatives of arithmetic combinations and compositions of powers of x, sine and cosine, exponential and logarithmic functions. Specifically, find derivatives of polynomial, tangent, and secant functions.
4.8 Use implicit differentiation to find derivatives of implicitly defined functions.
4.9 Use implicit differentiation to find derivatives of inverse functions. Specifically, find derivatives of inverse sine and inverse tangent.
4.10 Recognize conditions under which the derivative of a function fails to exist for functions given graphically or algebraically, including discontinuities, vertical tangents, and places where left and right hand derivatives differ (i.e., cusps and corners).

5. Applications of the Derivative

5.1 Demonstrate and explain the relationship between the graph of a function and its first and second derivatives .
- 5.1.1 Determine the sign of f '(x) on an interval by noting whether f(x) is increasing or decreasing on the interval, and vice versa.
- 5.1.2 Determine the sign of f "(x) on an interval from the concavity of f(x) on the interval, and vice versa.
- 5.1.3 Use f '(x) to find intervals on which f(x) is increasing and decreasing, and to find the relative extrema of f(x).
- 5.1.4 Use f "(x) to find intervals on which f(x) is concave up and concave down, and to find points of inflection on the graph of f(x).
- 5.1.5 Given a graph of f(x), sketch f '(x) and f "(x).
- 5.1.6 Given a graph of f '(x) sketch a possible graph of f(x).
- 5.1.7 Describe the shape of a graph using English, e.g., Increasing at an increasing rate; increasing at a decreasing rate; decreasing at an increasing rate; decreasing at a decreasing rate.

5.2 Given a position function for an object in motion, find its velocity and acceleration functions. Locate extreme positions and the times when they occur.
5.3 Use differentials to approximate the change in the dependent variable.
5.4 Solve problems in optimization.
5.5 Find slopes of curves at any point and equations of tangent lines at these points. Use the tangent line to f(x) at c to approximate f(x) near c.
5.6 Solve related rates problems.

6. Antiderivatives

6.1 Write and apply the definition of an indefinite integral to find antiderivatives of sums and differences of constant multiples of powers of x, sines, cosines, and exponential functions.

6.2 Use an initial condition to find a particular solution to the equation $y = f(x)dx$ to solve problems involving position, velocity and acceleration.

7. Integrals

7.1 Write the Riemann sum definition of the definite integral.

7.2 Use left and right sums to approximate the area under the graph of a function represented algebraically, graphically, or by a table of values.

- 7.2.1 Sketch and shade the areas corresponding to both left and right hand sums.
- 7.2.2 Represent the area of each rectangle in terms of its height and width.
- 7.2.3 Calculate left and right hand sums for small n without a calculator program.
- 7.2.4 Use a calculator program to calculate left and right hand sums for functions given algebraically.
- 7.2.5 Calculate the maximum error of left and right hand sum approximations for monotonic functions (i.e., those which increase on an entire interval or decrease on an entire interval.)
- 7.2.6 Find the smallest number of subdivisions necessary for left and right hand sum approximations of monotonic functions to have a given maximum error.

7.3 Write and apply the Mean Value Theorem for Definite Integrals and both versions of the Fundamental Theorem of Calculus.

- 7.3.1 Find the exact value of the definite integral of a function given algebraically.
- 7.3.2 Find the average value of a continuous function on an interval [a,b].
- 7.3.3 Use the second part of the Fundamental Theorem to represent a particular antiderivative, to evaluate the antiderivative for particular values of x, and to give a graphical analysis of the antiderivative using its first and second derivatives.
- 7.3.4 Given a graph of f(x) and a point on a particular antiderivative for f(x), sketch the graph of the particular antiderivative. Identify locations of relative extrema and points of inflection and estimate the value of the antiderivative at these locations by approximating the area under the graph of f(x).
- 7.3.5 Describe a definite integral as the amount of change in a function, whose rate of change is given in the integrand, over the interval of values given by the limits of integration.

7.4 Use rules of integration to evaluate definite integrals of sums and differences of constant multiples of powers of x, sines, cosines, and exponential functions.

7.5 Apply rules involving the limits of integration.

- 7.5.1 Given the value of a definite integral, find the value of the integral that results from reversing the limits of integration.
- 7.5.2 Find the value of a definite integral whose limits of integration are equal.
- 7.5.3 Given the value of the definite integral of a function from a to c and the value of the definite integral of the same function from c to b, find the value of the definite integral of the function from a to b.
- 7.5.4 Evaluate the definite integral of the absolute value of a function by rewriting it as a sum of integrals over intervals for which the sign of the function does not vary.

7.6 Use a definite integral to find the exact area under a curve.

7.7 Write distance, displacement, average speed and average velocity in terms of a definite integral involving a velocity function. Find or approximate the integral that gives distance or displacement for velocity functions given algebraically, graphically, and by a table of values.

7.8 Use the built-in numerical integration function of a graphing utility to approximate a definite integral.

7.9 Evaluate both indefinite and definite integrals using simple u-substitution to express an integrand as u^n (including n = -1), sin u, cos u, or e^u where u is a simple trigonometric or polynomial function of x.