MTH 212 CALCULUS III – Course Objectives

Students will be expected to demonstrate an understanding of Calculus beyond the manipulation of symbols, apply Calculus to practical problems and use current technology throughout the course. They will demonstrate their understanding of Calculus using three approaches – geometric, numerical, and algebraic. A comprehensive final exam testing the degree of mastery of the following course objectives is required.

1. Functions of many variables

- 1.1 Represent and evaluate functions of two variables graphically (contour maps), numerically (tables), and by formulas.
- 1.2 Interpret functions of more than one variable by holding all but one variable fixed.
- 1.3. Plot points and draw graphs of functions of two variables as surfaces in three dimensions using traces in planes parallel to the coordinate planes (i.e. cross sections obtained by holding all but one variable fixed).
- 1.4 Graph cross sections for a function given by its contour diagram.
- 1.5. Represent and recognize linear functions as tables of values, parallel, equally spaced contour lines, flat surfaces, and first degree equations.
- 1.6. Represent functions of three variables as level surfaces.

2. Vectors

- 2.1. Find the magnitude, direction and component form of displacement vectors.
- 2.2. Perform the following vector operations:
  - – addition and subtraction,
  - – scalar multiplication,
  - – dot product, geometric and component forms,
  - – cross product, geometric and component forms.
- 2.3. Use vector models for applications of velocity, force, work, finding angles between vectors, and projections.
- 2.4. Recognize, construct, and interpret equations of planes from tables, contour lines, 3 points, and a point and a normal vector.

3. Partial Derivatives

- 3.1. Find partial derivatives
  - – in direction of x, y (z, etc.)
  - – in any direction
  - – using numeric, graphic, and algebraic computations.
- 3.2. Interpret units and signs of partial derivatives.
- 3.3. Find tangent plane approximations and discuss local linearity for functions of two variables.
- 3.4. Find and interpret differentials for functions of two variables.
- 3.5. Compute gradients geometrically and algebraically.
- 3.6. Find maximum, minimum and zero rates of change and their directions using the gradient.
- 3.7. Give the gradient in component form.
- 3.8. Use the gradient to find directional derivatives.
- 3.9. Use chain rules to determine derivatives and partial derivatives for functions of several variables and to solve applied problems involving related rates of change.

3.10. Find Second Order Partial Derivatives given an algebraic representation of a function.
3.11. Find the signs of first and second order partial derivatives given a contour diagram for the function.
3.12. Determine whether or not a function satisfies a partial differential equation.
3.13. Find a Taylor Polynomial of degree two, i.e. a quadratic approximation, for a function of two variables.

4. Optimization

4.1. Find global and local extrema for a function given algebraically or via its contour diagram. Use the second derivative test to distinguish among local maxima, local minima, and saddle points.
4.2 Solve word problems involving optimization of functions of two or more variables, including maximizing profit, and least square approximations.
4.3 Solve problems in constrained optimization using the method of Lagrange multipliers, algebraically and using a contour diagram.

5. Multiple Integration

5.1 Use a Riemann Sum to approximate a double integral, where the Riemann Sum arises from an application, as total population given a density function over a two dimensional area (e.g. fox density map of England).
5.2 Interpret the two-variable integral as a volume under the graph of a function of two variables, a total amount of a function of two variables (when the integrand is density per area), and the volume over the region of integration at a constant height equal to the average value of the integrand on the region.
5.3 Sketch regions of integration and reverse the order of integration for double and triple integrals.
5.4 Set up and evaluate double and triple iterated integrals over 2- and 3- dimensional regions.
5.5. Set up and evaluate definite integrals in two dimensions using polar coordinates. Change from polar to rectangular coordinates and vice versa.

6. Parametric Curves

6.1 Describe the motion in the plane or in space of an object using parametric representation.
6.2 Find and use velocity and acceleration vectors for motion described parametrically.
6.3 Represent curves parametrically, implicitly and explicitly. Be able to convert from one form of representation to another.
6.4 Use the velocity vector to find the length of a curve.

7. Vector Fields

7.1 Give vector fields via a formula or sketch. Convert between these representations.
7.2 Recognize types of vector fields, including velocity fields, force fields, and gradient fields.
7.3 Draw flow lines in a vector field.
7.4 Show that a particular flow satisfies the differential equations for an associated vector field.

8. Line Integrals

8.1 Define the line integral as the sum of dot products of vector field and path elements. Determine the sign of a line integral give a sketch of the field and the path.

8.2 Apply the concept of line integral to work and circulation.

8.3 Compute line integrals given the component functions of the vector field by parameterizing the path (oriented curve).

8.4 Apply the fundamental theorem of calculus for line integrals using the gradient of a scalar function.

8.5 Know the definition and properties of conservative vector fields and their relationship to gradient fields.

8.6 Equate path independence with circulation free.

8.7 Discuss the fact that nonconservative fields are neither path independent nor circulation free.

8.8 Apply Green's Theorem to evaluate a line integral on a closed path.