# MTH 155 Objectives

Use problem-solving approaches to demonstrate strategies to investigate patterns and similarities among problems throughout this course. The spirit of approach is to develop more than one way to explain processes of problem solving. Mathematical understanding is achieved through experiment, discovery, conjecture, invention and reasoning. This is not a teaching methods course.

#### 1. Problem Solving

- 1.1. Describe, illustrate, and apply George Pólya's problem solving principles.
- 1.2. Build new mathematical knowledge through problem solving.
- 1.3. Apply and adapt a variety of appropriate strategies to solve problems.
- 1.4. Make and investigate mathematical conjectures.
- 1.5. Analyze and evaluate the mathematical thinking and strategies of others.

## 2. Sets

- 2.1. Create and use a variety of representations of sets and subsets to organize, record, and communicate mathematical ideas.
- 2.2. Translate among set representations to solve problems.
- 2.3. Model physical, social, and mathematical phenomena using sets.

## 3. Development of place-value number systems

- 3.1. Represent numbers and interpret numerals within historical additive number systems (including the Egyptian and Roman systems)
- 3.2. Represent numbers and interpret numerals within historical place-value systems (including the Babylonian and Indo-Arabic systems).
- 3.3. Compute sums and differences in historic additive and place-value systems.
- 3.4. Translate numerals among place-value systems using various bases.
- 3.5. Add, subtract, and multiply\* multi-digit numerals in non-decimal bases using traditional algorithms. (\*optional)
- 3.6. Demonstrate the use of physical representations to illustrate numerals in decimal and nondecimal based place value systems.

## 4. Whole number and integer arithmetic

- 4.1. Identify and give examples of the following properties:
  - a. whole number and integer addition (closure, commutative, associative, additive identity, and additive inverse properties).
  - b. whole number and integer multiplication (closure, commutative, associative, multiplicative identity, zero, and distributive properties).
- 4.2. Illustrate whole number addition using the set model and the number line model.
- 4.3. Illustrate whole number subtraction using the take-away, missing addend, comparison, and number line models.
- 4.4. Demonstrate whole number multiplication using the repeated addition, array, and Cartesian product models.
- 4.5. Demonstrate whole number division using the repeated subtraction, partition, and missing factor models.
- 4.6. Illustrate integer addition and subtraction using two-sided counters, the number line, and numerical patterns.

- 4.7. Demonstrate integer multiplication using the repeated addition, number line, and numerical pattern models.
- 4.8. Evaluate integer expressions using the appropriate order of operations and laws of exponents, both with and without the aid of a calculator.
- 4.9. Compute reasonable estimates of the values of integer expressions using mental arithmetic.
- 4.10. Illustrate multi-digit addition and subtraction computations using physical representations of place value.
- 4.11. Compute products of multi-digit numbers using both the lattice and traditional algorithms, and describe the relation between these algorithms and the array model.
- 4.12. Compute quotients and remainders of multi-digit numbers using both the scaffold and traditional algorithms, and describe the relation between these algorithms and the repeated subtraction model.

## 5. Elementary number theory

- 5.1. Identify unit, prime, and composite numbers.
- 5.2. List factors and multiples of a given number.
- 5.3. Build the prime power representation of a given number.
- 5.4. Apply divisibility tests to quickly determine if a number is divisible by 2, 3, 4, 5, 8, 9, 10, or 11.
- 5.5. Determine whether a given number is divisible by a power of ten.
- 5.6. Find the greatest common divisor and least common multiple for two numbers by:
  - a. listing their divisors or multiples
  - b. using their prime power representations
  - c. using Venn diagrams (representing each number as a set of primes)
  - d. using the Euclidean algorithm.

#### 6. Rational numbers

- 6.1. Identify and use properties of rational numbers including closure, commutative, associative, distributive, identities, and inverses.
- 6.2. Illustrate the traditional algorithms for fraction arithmetic using visual representations.
- 6.3. Perform operations on rational numbers as fractions, decimals, and percents.
- 6.4. Solve problems using ratios and proportional reasoning.
- 6.5. Convert between fraction and decimal representations for rational numbers.

#### 7. Real numbers

- 7.1. Identify and use the closure, commutative, associative, distributive, identities, inverses, and denseness properties for real numbers.
- 7.2. Identify irrational numbers in radical and decimal form.
- 7.3. Illustrate the traditional algorithms for real decimal arithmetic in terms of fraction and place value models.