<u>MTH 210 CALCULUS I</u> (Effective Fall 2016)

Students will be expected to demonstrate an understanding of Calculus beyond the manipulation of symbols, to use mathematically correct terminology and notation, and to apply the theory and methods of Calculus to solve a variety of different problems. A comprehensive departmental final exam testing the degree of mastery of the following course objectives is required. The use of integration tables, formula sheets, graphing calculators, and calculators with computer algebra systems is not permitted on the final exam.

1. Limits and Continuity
   - 1.1. Estimate the value of a one- or two-sided limit by creating a table of values.
   - 1.2. Given the graph of a function, determine one-sided limits, two-sided limits, and function values at specified points, or indicate that they do not exist or are undefined.
   - 1.3. Apply properties of limits.
   - 1.4. Evaluate one and two-sided limits by direct substitution, when possible.
   - 1.5. Analytically evaluate limits involving indeterminate forms of type 0/0 by using methods such as factoring and canceling, multiplying by a conjugate, and multiplying by a least common denominator.
   - 1.6. Analytically evaluate limits of piecewise-defined functions.
   - 1.7. Analytically evaluate limits of absolute value functions.
   - 1.8. Analytically evaluate limits that can be put in the form $\lim\limits_{x\to 0}\dfrac{\sin(ax)}{ax}$ and $\lim\limits_{x\to 0}\dfrac{1-\cos(ax)}{ax}$, where a is a nonzero constant.
   - 1.9. Given a function that is unbounded at c, analytically determine $\lim\limits_{x\to c^-} f(x)$, $\lim\limits_{x\to c^+} f(x)$, and $\lim\limits_{x\to c} f(x)$, or indicate that they do not exist.
   - 1.10. Determine the vertical asymptotes of a function (if any), and describe the behavior of the function at each vertical asymptote using infinite limits.
   - 1.11. Analytically evaluate limits of the form $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$, and use the results to determine the horizontal asymptotes of the function (if any).
   - 1.12. Analytically evaluate limits involving indeterminate forms of type ∞/∞ where the numerator and denominator contain polynomial functions, radical functions, or sums of exponential functions.
   - 1.13. State that a function f is continuous at a point c if and only if all of the following conditions are satisfied: $f(c)$ is defined, $\lim\limits_{x\to c} f(x)$ exists, and $\lim\limits_{x\to c} f(x) = f(c)$.
   - 1.14. Given a graph of a function, determine the values of x at which the function is discontinuous.
   - 1.15. Given a function defined by a formula, determine the values of x at which the function is discontinuous.
   - 1.16. Given a piecewise-defined function, apply the continuity checklist (the three step test for continuity) to determine whether or not the function is continuous at a specified point.

2. Techniques of Differentiation
    2.1. State the limit definition of the derivative.
    2.2. Use the limit definition of the derivative to determine the derivative of a polynomial function, a radical function, and a rational function.
    2.3. Use any of the following notations to represent the derivative of a function $y = f(x)$:

    $y', f'(x), \dfrac{dy}{dx}, \dfrac{d}{dx}\left(f(x)\right)$.
    2.4. Interpret the value of the derivative of a function at a point as the slope of the tangent line to the graph of the function at that point.
    2.5. Determine an equation of the tangent line to the graph of a function at a specified x-value.
    2.6. Determine the values of x at which a function is not differentiable.
    2.7. Determine the derivative of each of the following functions:

    x^n (where n is any real number), $\sin x,\ \cos x,\ \tan x,\ \sec x,\ \csc x,\ \cot x,$

    $e^x,\ a^x,\ \ln x,\ \log_a x,\ \sin^{-1} x,\ \tan^{-1} x,\ \sec^{-1} x$.
    2.8. Apply the Constant Multiple Rule, the Sum Rule, the Difference Rule, the Product Rule, the Quotient Rule, and the Chain Rule to differentiate combinations and compositions of any of the functions listed in objective 2.7.
    2.9. Determine higher order derivatives.
    2.10. Use any of the following notations to represent higher order derivatives of $y = f(x)$:

    $y^{(n)}, f^{(n)}(x), \dfrac{d^n y}{dx^n}, \dfrac{d^n}{dx^n}\left(f(x)\right)$. Second and third order derivatives may also be denoted using the double prime or triple prime notation.
    2.11. Use implicit differentiation to determine the derivative of a function defined implicitly by a relation.
    2.12. Determine the equation of the tangent line to an implicitly defined relation at a specified point.
    2.13. Use logarithmic differentiation to determine the derivative of a function.

3. Applications of Derivatives
    3.1. Calculate the average rate of change of a function on an interval, and the instantaneous rate of change of a function at a point.
    3.2. Given a position function for an object moving along a line, determine the average velocity of the object on a time interval, including correct units.
    3.3. In a variety of applied contexts, interpret the value of the derivative at a point as the instantaneous rate of change of a quantity at that point, including correct units, and use the sign of the derivative to tell if the quantity is increasing or decreasing at that point.
    3.4. Given a position function for an object moving along a line, determine the functions that give the instantaneous velocity, acceleration, and speed of the object, and calculate the instantaneous velocity, acceleration, and speed at specified times, including correct units.

3.5. Solve related rate problems, including correct units.
3.6. Determine the critical points of a function.
3.7. Determine the absolute (global) maximum and absolute (global) minimum value of a continuous function on a closed interval.
3.8. Use the first derivative to determine the open intervals on which a function is increasing and decreasing, and state the coordinates of any local (relative) extrema.
3.9. Use the second derivative to determine the open intervals on which a function is concave up and concave down, and state the coordinates of any points of inflection.
3.10. Use the information obtained from the first and second derivatives to sketch the graph of a function, clearly showing any local (relative) extrema, points of inflection, intervals of concavity, and vertical or horizontal asymptotes.
3.11. Given the graph of a function, sketch a possible graph of its derivative.
3.12. Given the graph of the first derivative of a function and a specified point that the function passes through, sketch a possible graph of the function.
3.13. Given a list of conditions satisfied by a function and its first and/or second derivatives, sketch a possible graph of the function.
3.14. Solve optimization (max-min) problems, including correct units.
3.15. Given a position function for an object moving along a line, determine its extreme positions and the times at which they occur, including correct units.
3.16. State and apply the Mean Value Theorem for Derivatives.
3.17. Use a tangent line as a linear approximation of a function near a point.
3.18. Determine the differential of a function.
3.19. Given a function $y = f(x)$, determine dy and Δy for specified values of x and Δx.
3.20. Use differentials to approximate the change in the dependent variable in a function.

4. Integration
4.1. Determine the antiderivative of each of the following functions:
x^n (including $n = -1$), $\sin x$, $\cos x$, $\sec^2 x$, $\csc^2 x$, $\sec x \tan x$, $\csc x \cot x$, e^x, a^x.
4.2. Use rules of integration to determine indefinite integrals of linear combinations of any of the functions listed in objective 4.1.
4.3. Determine the general solution of a differential equation of the form $y' = f(x)$, as well as a particular solution satisfying a specified initial condition.
4.4. Given the acceleration function and initial conditions for an object moving along a line, determine the velocity function and the position function for the object.
4.5. Use integration by u-substitution to determine indefinite integrals where the integrand can be transformed into a constant multiple of any of the functions listed in objective 4.1.
4.6. Sketch the graph of a function, and draw in the rectangles used for either a Left- or Right-Hand Riemann Sum approximation of a definite integral of that function.
4.7. Calculate Left- and Right-Hand Riemann Sum approximations of a definite integral of a function given by a formula, a graph, or a table of values.

4.8. Given a function that is monotonic (either increasing or decreasing) on an interval, determine whether the Left- and Right-Hand Riemann Sums are over- or under-estimates of the actual value of the definite integral.

4.9. Determine the maximum error in using either the Left- or Right-Hand Riemann Sum to approximate a definite integral of a monotonic function.

4.10. Given a monotonic function, determine the minimum number of subdivisions for which the Left- and Right-Hand Riemann Sums will differ by less than a given amount.

4.11. State the definition of a definite integral as a limit of a Riemann Sum.

4.12. Apply properties of definite integrals, including each of the following: $\int_a^a f(x)dx = 0$, $\int_b^a f(x)dx = -\int_a^b f(x)dx$, and $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

4.13. Interpret the value of a definite integral as the net area bounded by the graph of a function and the x-axis on a specified interval.

4.14. State both parts of the Fundamental Theorem of Calculus.

4.15. Use the Fundamental Theorem of Calculus to determine the derivative of a function of the form $F(x) = \int_a^{g(x)} f(t)dt$, including the case where properties of definite integrals must be used to rewrite the function in the form required to apply the theorem.

4.16. Given a function of the form $F(x) = \int_a^x f(t)dt$, determine the critical points of F, the open intervals on which F is increasing and decreasing, the open intervals on which F is concave up and concave down, and the x-coordinates of any local (relative) extrema and points of inflection of F.

4.17. Use the Fundamental Theorem of Calculus to determine the exact value of a definite integral of a linear combination of any of the functions listed in objective 4.1.

4.18. Evaluate a definite integral of an absolute value function by writing the absolute value function as a piecewise function and applying properties of definite integrals.

4.19. Use integration by u-substitution to determine definite integrals where the integrand can be transformed into a constant multiple of any of the functions listed in objective 4.1.

4.20. Determine the exact area bounded by the graph of a continuous function and the x-axis over a specified interval.

4.21. Determine the exact average value of a continuous function on a closed interval.

4.22. In a variety of applied contexts, interpret the definite integral of a function over a specified interval as the net change in the function's antiderivative over the interval, including correct units.

4.23. Given a velocity function for an object moving along a line, use definite integrals to determine the displacement, total distance traveled, average velocity, and average speed of the object over a specified time interval, including correct units.