<u>MTH 211 CALCULUS II</u> (Effective Fall 2016)

Students will be expected to demonstrate an understanding of Calculus beyond the manipulation of symbols, to use mathematically correct terminology and notation, and to apply the theory and methods of Calculus to solve a variety of different problems. A comprehensive departmental final exam testing the degree of mastery of the following course objectives is required. The use of integration tables, formula sheets, and calculators with computer algebra systems is not permitted on the final exam.

1. Integration Techniques
   - 1.1. Review finding indefinite and definite integrals of linear combinations of x^n (including the case where $n = -1$), $\sin x$, $\cos x$, $\sec x \tan x$, $\csc x \cot x$, $\sec^2 x$, $\csc^2 x$, e^x, and a^x.
   - 1.2. Find antiderivatives of $\tan x$, $\cot x$, $\sec x$, $\csc x$, $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{a^2-x^2}}$, and $\frac{1}{x\sqrt{x^2-a^2}}$ $(a > 0)$.
   - 1.3. Use integration by substitution to find integrals where the integrand can be transformed into any of the basic functions listed in objectives 1.1 and 1.2.
   - 1.4. Find integrals that require transforming the integrand into a form that can be integrated by using basic trig identities, or algebraic techniques such as completing the square and long division.
   - 1.5. Find integrals by using integration by parts.
   - 1.6. Find integrals by using partial fraction decomposition.
   - 1.7. Integrate functions involving products of powers of sine and powers of cosine.
   - 1.8. Integrate functions involving products of powers of secant and powers of tangent.
   - 1.9. Integrate using trigonometric substitutions of the form $u = a \sin\theta$, $u = a \tan\theta$, and $u = a \sec\theta$.
   - 1.10. Use the following numerical techniques of integration to approximate a definite integral for a function given algebraically, graphically, or as a table of values:
     - 1.10.1. Midpoint rule
     - 1.10.2. Trapezoid rule
     - 1.10.3. Simpson's rule
   - 1.11. Use concavity to determine whether the Midpoint and Trapezoid rules are over- or under-estimates of a definite integral.

2. Applications of Integration

   Carry out the following process for each of the applications listed in objectives 2.1 – 2.3:
   - Write an expression that represents the quantity being calculated for the i^{th} element of a partition.
   - Construct a Riemann Sum that approximates the total quantity being calculated.
   - Express the exact value of the total quantity being calculated as the limit of a Riemann Sum.
   - Write and evaluate a definite integral to find the exact value of the total quantity being calculated.
   - 2.1. Find the area between two curves.
   - 2.2. Find volumes of solids of revolution using disks, washers, and shells.
   - 2.3. Find the work done in moving an object along a straight line by a force in the direction of motion.

3. First Order Differential Equations
   - 3.1. Find the general solution of a separable differential equation by using the separation of variables technique.
   - 3.2. Given a separable differential equation with an initial condition, find the particular solution of the initial-value problem.
   - 3.3. Given the slope field for a differential equation of the form $\frac{dy}{dx} = f(x, y)$, sketch the graph of a particular solution of the differential equation which satisfies a given initial condition.
   - 3.4. Translate a verbal description of a rate of change into a separable differential equation, and then find the particular solution that satisfies given initial conditions. Examples should include, but not be limited to, modeling exponential growth and decay.

4. Limits with Indeterminate Forms
   - 4.1. Use L'Hopital's Rule to evaluate limits involving indeterminate forms of the type $0/0$ and ∞/∞.
   - 4.2. Apply algebraic techniques to reduce limits involving indeterminate forms of type $0 \cdot \infty$, and $\infty - \infty$ to forms for which L'Hopital's Rule can be applied.
   - 4.3. Use logarithmic techniques to reduce limits involving indeterminate forms of type $0^0, \infty^0$, and 1^∞ to forms for which L'Hopital's Rule can be applied.

5. Improper Integrals
   - 5.1. Evaluate improper integrals with infinite limits of integration.
   - 5.2. Evaluate improper integrals with discontinuous integrands.
6. Infinite Series
   - 6.1. Determine if a sequence converges or diverges, and if it converges, find its limit.
   - 6.2. Given a series, calculate a sequence of its partial sums and construct an expression for the n^{th} partial sum when possible.
   - 6.3. Determine the convergence or divergence of a telescoping series by evaluating the limit of the n^{th} partial sum, and give its sum if it converges.
   - 6.4. Recognize geometric series, determine their convergence or divergence, and find their sum if convergent.
   - 6.5. Use the n^{th} Term Test for Divergence.

6.6. Interpret the terms of a series as areas of rectangles of width one and height equal to the value of the term. Interpret the Integral Test as a comparison of an improper integral to its upper- or lower-sum.
6.7. Use the Integral Test to determine the convergence or divergence of a series when applicable. Use an appropriate integral to give an upper bound for the error in using a partial sum to approximate the sum of the series.
6.8. Apply the p-Series Test to determine the convergence or divergence of a p-series.
6.9. Use the Direct Comparison Test to determine the convergence or divergence of a series.
6.10. Recognize the *Harmonic Series* and know that it diverges.
6.11. Recognize an alternating series, and apply the Alternating Series Test when applicable.
6.12. Use a partial sum to approximate the sum of an alternating series whose terms decrease in magnitude and approach zero. Give the magnitude of the first neglected term as an upper bound for the error of the approximation.
6.13. Use the Ratio Test to determine the convergence or divergence of a series.
6.14. Use the Root Test to determine the convergence or divergence of a series.
6.15. Calculate Taylor Polynomial approximations. Discuss the accuracy of the approximations by comparing graphs and by using the Lagrange form of the Taylor remainder.
6.16. Determine the radius and interval of convergence for a power series.
6.17. Perform substitution, algebraic manipulation, and term-by-term differentiation and integration of known power series to form new series. The known power series should include the power series centered at 0 for: $\sin x$, $\cos x$, e^x, and $\frac{1}{1-x}$.
6.18. Construct Taylor Series expansions of functions and examine their intervals of convergence.
6.19. Construct a Taylor Polynomial for a function, and use it to approximate a definite integral.

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