<u>MTH 212 CALCULUS III</u> (Effective Summer 2019)

Students will be expected to demonstrate an understanding of Calculus beyond the manipulation of symbols, to use mathematically correct terminology and notation, and to apply the theory and methods of Calculus to solve a variety of different problems. A comprehensive final exam testing the degree of mastery of the following course objectives is required.

1. Vector Algebra

    1.1. Express vectors in component form and as a linear combination of the standard unit vectors **i**, **j**, and **k**.
    1.2. Determine the norm of a vector.
    1.3. Determine the components of a vector given its initial and terminal point.
    1.4. Determine the components of a vector in the plane given its norm and the angle it makes with the positive x-axis.
    1.5. Perform vector operations, including addition, subtraction, scalar multiplication, dot product, and cross product.
    1.6. Analyze the algebraic and geometric properties of vector operations.
    1.7. Determine a unit vector in the direction of a nonzero vector.
    1.8. Determine the angle between two nonzero vectors.
    1.9. Determine if two nonzero vectors are parallel, orthogonal, or neither.
    1.10. Given two nonzero vectors **u** and **v**, determine the vector projection of **u** onto **v**, and the vector component of **u** orthogonal to **v**.
    1.11. Construct parametric equations for lines in space satisfying specified conditions.
    1.12. Construct equations for planes in space satisfying specified conditions.
    1.13. Use vectors to solve applied problems involving basic physics concepts such as velocity, force, and work.

2. Vector-Valued Functions of a Single Variable

    2.1. Construct parametric equations for curves in the plane and in space.
    2.2. Represent a parameterized curve using a vector-valued function.
    2.3. Sketch the curve represented by a vector-valued function.
    2.4. Convert between explicit, implicit, and parametric representations of curves.
    2.5. Differentiate vector-valued functions.
    2.6. Integrate vector-valued functions.
    2.7. For problems involving motion of a particle along a curve represented by a vector-valued function, determine the velocity vector, acceleration vector, and speed of the particle at a given time.
    2.8. Determine the arc length of a curve represented by a vector-valued function.

3. Real-Valued Functions of Several Variables

    3.1. Evaluate functions of several variables.
    3.2. Determine the domain and range for functions of several variables.
    3.3. Sketch level curves for functions of two variables.
    3.4. Represent functions of two variables in various forms such as formulas, tables, graphs of surfaces in space, and contour maps.
    3.5. Represent functions of three variables as level surfaces.
    3.6. Analyze functions of several variables by holding all but one variable fixed.

4. Multivariable Differential Calculus

    4.1. Determine partial derivatives for functions of several variables.
    4.2. Determine second order partial derivatives for functions of several variables.
    4.3. Interpret the signs and units of partial derivatives for application problems.
    4.4. Determine whether a function of several variables satisfies a given partial differential equation.
    4.5. Determine the total differential for a function of several variables.
    4.6. Use the total differential for a function of several variables to approximate the change in the dependent variable given the change in each of the independent variables.
    4.7. Apply the Chain Rule to compute derivatives and partial derivatives.
    4.8. Use the Chain Rule to solve related rate problems.
    4.9. Determine the gradient vector for a function of several variables.
    4.10. Determine the directional derivative of a function of several variables in the direction of a given vector.
    4.11. Use the gradient vector for a function of several variables to determine the direction of maximum increase and the maximum value of the directional derivative of the function at a point.
    4.12. Explain that the gradient vector for a function of two variables at a point is orthogonal to the level curve of the function through that point.
    4.13. Construct an equation for the tangent plane to the graph of a function of two variables at a point, and use it as a linear approximation of the function near the point.
    4.14. Locate the critical points for a function of two variables.
    4.15. Apply the Second Partials Test to classify the critical points of a function of two variables as relative maxima, relative minima, or saddle points.
    4.16. Use the method of Lagrange Multipliers to determine the maximum and minimum value of a function of several variables subject to a given constraint.
    4.17. Determine the absolute maximum and absolute minimum value of a continuous function of two variables defined on a closed and bounded region in the *xy*-plane.
    4.18. Solve applied optimization problems.

5. Multivariable Integral Calculus

    5.1. Define a double integral as a limit of a Riemann Sum for a function of two variables.
    5.2. Evaluate double integrals.
    5.3. Sketch the region of integration in the *xy*-plane for a double integral.
    5.4. Reverse the order of integration for a double integral.
    5.5. Use a double integral to determine the area of a closed and bounded region in the *xy*-plane.
    5.6. Use a double integral to determine the volume of the solid region that lies below the graph of a non-negative continuous function of two variables and above a closed and bounded region in the *xy*-plane.
    5.7. Determine the average value of a continuous function of two variables on a closed and bounded region in the *xy*-plane.
    5.8. When appropriate, convert a double integral from rectangular to polar coordinates, and evaluate the integral.
    5.9. Evaluate triple integrals.
    5.10. Change the order of integration for a triple integral.
    5.11. Use a triple integral to determine the volume of a solid region in space with given boundaries.
    5.12. Given a density function, use a double or triple integral to determine a total amount (for example, total population or total mass).

6. Vector Fields and Line Integrals

    6.1. Sketch representative vectors in a given vector field.
    6.2. Determine whether a vector field is conservative or non-conservative.
    6.3. Construct a potential function for a conservative vector field.
    6.4. Evaluate a line integral of a vector field over a given curve.
    6.5. Interpret a line integral as the work done by a force field on an object moving along a curve.
    6.6. Explain that line integrals of conservative vector fields are independent of path.
    6.7. Explain that conservative vector fields are circulation free, or equivalently, that line integrals of the vector field over closed curves must equal zero.
    6.8. Explain that for non-conservative vector fields, line integrals are not independent of path and line integrals over closed curves will not necessarily equal zero.
    6.9. Use the Fundamental Theorem of Line Integrals to evaluate a line integral of a conservative vector field over a given curve.
    6.10. Use Green's Theorem to evaluate a line integral of a vector field over a closed curve.