<u>MTH 225 DIFFERENTIAL EQUATIONS</u> (Effective Fall ‘14) Last modified 11/8/13

1. General Objectives:
   - 1.1 Use mathematically correct notation when writing and solving differential equations.
   - 1.2 Make appropriate use of mathematical terminology.
   - 1.3 Classify differential equations as:
     - a. First order, second order, etc…
     - b. Linear or Nonlinear
     - c. Separable or not separable
     - d. Autonomous or not autonomous
     - e. Homogeneous or Nonhomogeneous
   - 1.4 Understand and apply various theorems.

2. First Order Differential Equations
   - 2.1 Analytical Methods: Obtain both general solutions and particular solutions to initial value problems.
     - 2.1.1 Solve separable differential equations using separation of variables.
     - 2.1.2 Solve linear differential equations using integrating factors.
     - 2.1.3 Solve homogeneous differential equations by applying an appropriate substitution to transform it into a separable equation.
     - 2.1.4 Solve Bernoulli differential equations by applying an appropriate substitution to transform it into a linear equation.
     - 2.1.5 Indicate the interval of definition for a particular solution to a differential equation.
     - 2.1.6 Identify any singular solutions of a differential equation (i.e. solutions not included in a general solution).

   - 2.2 Numerical and Qualitative Methods:
     - 2.2.1 Use Euler’s Method to estimate a particular solution of a differential equation.
     - 2.2.2 Use a slope/direction field for a differential equation to describe the qualitative behavior of its solutions based on various initial conditions.
     - 2.2.3 Create a phase line for an autonomous differential equation.
     - 2.2.4 Use both phase lines and slope/direction fields to determine qualitative behavior (such as long term behavior) of the solutions of a differential equation.
     - 2.2.5 Obtain and classify the equilibrium-/critical-points of an autonomous differential equation (stable, unstable, semistable, source, sink, node).

   - 2.3 Applications and Modeling
     - 2.3.1 Translate a verbal statement into a differential equation.
     - 2.3.2 Model and solve a variety of application problems. The variety of problems chosen should reflect the various methods of solving differential equations taught in the course. Possible examples include electric circuits, compartmental analysis, growth/decay models, chemical mixtures, and orthogonal trajectories.

3. Systems of First Order Differential Equations

3.1 Solve a system of two first order linear differential equations with constant coefficients using either elimination methods or matrix methods.

3.2 Use a phase plane to determine qualitative behavior of the solutions of an autonomous system of first order differential equations.

3.3 Obtain and classify the equilibrium-/critical-points of an autonomous system of first order differential equations (stable, unstable, asymptotically stable).

3.4 Use a system of two first order linear differential equations to model, analyze, and solve an application problem. Possible examples include population dynamics and harmonic oscillators (spring/mass systems, series circuits).

4. Second Order Differential Equations

4.1 General Theory of Linear Differential Equations

4.1.1 Use the superposition principal to recognize that any linear combination of solutions to a homogeneous linear differential equation is also a solution. Recognize that this is not true for nonhomogenous and nonlinear differential equations.

4.1.2 Use the superposition principal to recognize that the sum of a solution to a homogeneous linear differential equation and a solution of an associated nonhomogeneous equation is a solution of the associated nonhomogeneous equation.

4.1.3 Recognize that the general solution of a second order homogenous linear differential equation can be expressed as an arbitrary linear combination of any two linearly independent solutions.

4.1.4 Recognize that the general solution of a second order nonhomogenous linear differential equation can be expressed as the sum of the general solution of the associated homogenous equation and any particular solution of the nonhomogeneous equation.

4.2 Analytical Methods: Obtain both general solutions and particular solutions to initial value problems.

4.2.1 When applicable, apply a substitution such as $u = y'$ to reduce a second order differential equation to a first order differential equation and then solve the resulting differential equation using an appropriate first order method.

4.2.2 Solve a second order homogeneous linear differential equation with constant coefficients using an auxiliary equation.

4.2.3 Use reduction of order to obtain a second linearly independent solution of a second order homogeneous linear differential equation, given one solution.

4.2.4 Solve a second order nonhomogeneous linear differential equation with constant coefficients by the method of undetermined coefficients.

4.2.5 Solve a second order nonhomogeneous linear differential equation with constant coefficients by the method of variation of parameters.

4.2.6 Obtain a power series solution centered about an ordinary point for a differential equation with polynomial coefficients. Along with the solution, include a lower bound for the radius of convergence of the power series solution.

4.3 Applications and Modeling

4.3.1 Model and analyze (qualitatively, quantitatively, and analytically) the behavior of a spring-mass system using a differential equation and its solutions.

4.3.2 Use Kirchhoff's Law to model and analyze (qualitatively, quantitatively, and analytically) the behavior of a series circuit using a differential equation and its solutions.

5. The Laplace Transform

5.1 Use the integral definition of the Laplace Transform to obtain the Laplace Transform of a variety of basic functions including piecewise defined functions.

5.2 Use the linearity of the Laplace Transform along with transform rules for functions of the form: $1, t^n\ (n \in \mathbb{Z}, n \geq 0), e^{kt}, \sin(kt), \cos(kt), \sinh(kt), \cosh(kt)$, and the Dirac Delta Function, $\delta(t)$, to obtain the Laplace Transform of any linear combination of the abovementioned functions as well as the corresponding inverse Laplace Transforms.

5.3 Apply the Shifting/Translation Theorems for translations in the s- and t-domain.

5.4 Obtain the Laplace Transform of a derivative, an integral, and a periodic function using an established transform rule.

5.5 Express the inverse transform of a product of two functions in the s-domain as the convolution of appropriate functions in the t-domain.

5.7 Solve first and second order initial value problems using the Laplace Transform.