MTH 230 Linear Algebra

In this course, students should demonstrate proficiency in performing matrix operations by hand. After that proficiency is established, technology may be used to perform matrix operations, at the discretion of the instructor.

1. General Objectives: Throughout the course, students will be expected to demonstrate their understanding of Linear Algebra by being able to do each of the following:
   - 1.1 Use mathematically correct terminology and notation.
   - 1.2 State precise definitions.
   - 1.3 Apply theorems to solve a variety of problems.
   - 1.4 State conditions which are equivalent to a square matrix being invertible.
   - 1.5 Construct simple proofs involving basic definitions and concepts.

2. Linear Systems and Matrices
   - 2.1 Determine if a matrix is in row echelon or reduced row echelon form.
   - 2.2 Write the augmented matrix of a linear system of equations.
   - 2.3 Solve a linear system of equations by using Gaussian elimination with back substitution or Gauss Jordan elimination.
   - 2.4 Identify which variables in a linear system are leading variables and which are free variables.
   - 2.5 Perform scalar multiplication, matrix addition, matrix subtraction, and matrix multiplication.
   - 2.6 Determine the size of a matrix product given the size of each matrix, or state that the product is undefined.
   - 2.7 Find the transpose of a matrix.
   - 2.8 Analyze the properties of matrix operations.
   - 2.9 Express a matrix-vector product as a linear combination of the columns of the matrix.
   - 2.10 Find an elementary matrix associated with a given elementary row operation, and determine the inverse of the elementary matrix.
   - 2.11 Find the inverse of a square matrix, or determine that the inverse does not exist.
   - 2.12 Analyze the properties of matrix inverses.
   - 2.13 Write a linear system of equations in matrix-vector form, and solve it using inverses in the case where the coefficient matrix is square and invertible.
   - 2.14 Analyze the properties of diagonal, triangular, and symmetric matrices.

3. Determinants
   - 3.1 Find the determinant of a square matrix by using cofactor expansion.
   - 3.2 Find the determinant of a square matrix by using row reduction.
   - 3.3 Analyze the properties of determinants.

3.4 Apply Cramer’s Rule to solve a linear system of equations, when applicable.

4. Vector Spaces
   - 4.1 State the axioms for a vector space.
   - 4.2 Determine if a set of vectors with specified operations for addition and scalar multiplication is a vector space.
   - 4.3 Determine if a subset of a vector space is a subspace.
   - 4.4 Determine if a vector is an element of the span of a set of vectors.
   - 4.5 Express a vector as a linear combination of a set of vectors, or determine that doing so is impossible.
   - 4.6 Determine if a set of vectors is linearly independent or linearly dependent.
   - 4.7 Determine if a set of vectors forms a basis for a vector space.
   - 4.8 Find the coordinates of a vector relative to a given basis for the vector space.
   - 4.9 Determine the dimension of a vector space.
   - 4.10 Find and use the transition matrix from one basis to another basis.
   - 4.11 Find bases for the row space, column space, and null space of a matrix.
   - 4.12 Determine the rank and nullity of a matrix.
   - 4.13 Given the size and rank of a matrix, determine the dimensions of the row space, column space, and null space of the matrix, as well as the null space of the transpose of the matrix.

5. Inner Product Spaces
   - 5.1 State the axioms for an inner product space.
   - 5.2 Determine if a given function defines an inner product on a vector space.
   - 5.3 Calculate inner products, norms, and distances relative to a given inner product.
   - 5.4 Analyze the properties of inner products, norms, and distances.
   - 5.5 State and apply the Cauchy-Schwarz Inequality.
   - 5.6 Determine if two vectors are orthogonal relative to a given inner product.
   - 5.7 Determine if a given set of vectors is an orthogonal set or an orthonormal set.
   - 5.8 Find the coordinates of a vector relative to an orthogonal or orthonormal basis.
   - 5.9 Given an orthogonal or orthonormal basis for a subspace, find the orthogonal projection of a vector onto the subspace, and the component of the vector orthogonal to the subspace.
   - 5.10 Apply the Gram-Schmidt process to transform a basis into an orthonormal basis.

6. Eigenvalues and Eigenvectors
   - 6.1 State the definition of an eigenvalue and an associated eigenvector of a square matrix.
   - 6.2 Find the characteristic polynomial for a square matrix.

6.3 Solve the characteristic equation to find the real eigenvalues of a square matrix.
6.4 Find a basis for each real eigenspace of a square matrix.
6.5 Find the eigenvalues of a triangular matrix by inspection.
6.6 Determine if a square matrix is diagonalizable, and if yes, find a matrix that will diagonalize it.
6.7 Determine if a square matrix is orthogonal.
6.8 Analyze the properties of orthogonal matrices.
6.9 Given a symmetric matrix, find a matrix that will orthogonally diagonalize it.

7. Linear Transformations
7.1 State the definition of a linear transformation.
7.2 Determine if a given function is a linear transformation.
7.3 Analyze the properties of linear transformations.
7.4 Find and use the standard matrix of a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.
7.5 Find the standard matrices for simple geometric transformations such as reflections, rotations, and projections.
7.6 Find bases for the kernel and range of a linear transformation.
7.7 Determine the rank and nullity of a linear transformation.
7.8 Determine if a linear transformation is one-to-one and/or onto.
7.9 Explain why any real finite dimensional vector space of dimension *n* is isomorphic to $\mathbb{R}^n$.
7.10 Determine if a linear operator is invertible, and if yes, find the standard matrix for its inverse.
7.11 Find the standard matrix for the composition of two linear transformations.

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