MTH 260–Probability and Statistics I
COURSE OBJECTIVES

1. Probability
    a. Probability and Inference
        i. Compute and graph relative frequencies.
    b. A Review of Set Notation
        i. Write and use expressions and Venn diagrams involving unions, intersections, and complements.
        ii. Define prescribed subsets of a given set according to given conditions.
        iii. List points in prescribed subsets of a given set.
    c. A Probabilistic Model for an Experiment: The Discrete Case
        i. State the following theorems / define the terms, and apply them to examples.
            - Experiment
            - simple event
            - sample space
            - discrete sample space
            - event
            - probability axioms
            - DeMorgan's Laws
        ii. Given an experiment:
            - List the sample points (i.e. define the sample space).
            - Assign a reasonable probability to each sample point.
            - Using the sample space, find prescribed probabilities using terms like "at least," "at most," and "exactly."
            - Using the sample space, find probabilities defined by expressions such as $P(A), P(B), P(A \cup B), P(A \cap B), P(\bar{A} \cup B)$.
    d. Calculating the Probability of an Event: The Sample-Point Method
        i. Outline the following steps (The Sample-Point Method) and apply them to an example to find the probability of an event.
            - Define the experiment and clearly determine how to describe one simple event.
            - List the simple events associated with the experiment and test each to make certain that it cannot be decomposed.
            - Assign reasonable probabilities to the sample points in the sample space, making certain that the probabilities are nonnegative and sum to 1.
            - Find the probability of an event by summing the probabilities of the sample points in the event.
    e. Tools for Counting Sample Points
        i. State the following theorems / define the terms, and apply them to examples.
            - Multiplication Principle of counting
            - Number of ways of filling r positions with n distinct objects
            - Number of ways of partitioning n distinct objects into k distinct groups containing $n_1, n_2, \dots, n_k$ objects, respectively, where each object appears in exactly one group and the sum of the n_i is n.
            - Number of combinations of n objects taken r at a time

- Number of unordered subsets of size r chosen (without replacement) from n available objects

f. Conditional Probability and the Independence of Events
    i. State the following theorems / define the terms, and apply them to examples.
        - Conditional probability of an event
        - Independence of events

g. Two Laws of Probability
    i. State the following theorems / define the terms, and apply them to examples.
        - The Multiplicative Law of Probability (for the intersection of two events)
        - The Addition Law of Probability (for the union of two events)

h. Calculating the Probability of an Event: The Event-Composition Method
    i. Give a summary of the event-composition method including:
        - Define the experiment.
        - Visualize the nature of the sample points.
        - Write a probability expression for the event of interest.
        - Apply the additive and multiplicative laws of probability to the compositions of interest.

i. The Law of Total Probability and Bayes' Rule
    i. State the following theorems / define the terms, and apply them to examples.
        - Partition of a sample space
        - For a partition $\{B_1, B_2, \ldots, B_k\}$ of a sample space S such that $P(B_i) > 0$ for $i = 1, \ldots, k$; for any event A, the probability $P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$.
        - Bayes' Rule

2. Discrete Random Variables and Their Probability Distributions
    a. The Probability Distribution of a Discrete Random Variable
        i. State the following theorems / define the terms, and apply them to examples.
            - Discrete random variable
            - $P(Y = y)$ is the sum of the probabilities of all sample points in a sample space S that are assigned to the value y.
            - Probability distribution of a discrete random variable as a formula, table, or graph
            - Criteria for a discrete probability distribution
    b. The Expected Value of a Random Variable or a Function of a Random Variable
        i. State the following theorems / define the terms, and apply them to examples.
            - Expected value of a discrete random variable
            - Expected value of $g(Y)$ for a discrete random variable Y and a real-valued function g.
            - Variance and standard deviation (in terms of expected value)
            - Expected value of a constant
            - Linearity of expected value
    c. The Binomial Distribution
        i. State the following theorems / define the terms, and apply them to examples.

- Binomial experiment properties
- Binomial distribution based on n trials
- Mean and variance of a binomial random variable

d. The Geometric Probability Distribution
  i. State the following theorems / define the terms, and apply them to examples.
    - Geometric probability distribution of a random variable
    - Mean and variance of a geometric distribution

e. The Hypergeometric Probability Distribution
  i. State the following theorems / define the terms, and apply them to examples.
    - Hypergeometric probability distribution
    - Mean and variance of hypergeometric variables

f. Poisson Probability Distribution
  i. State the following theorems / define the terms, and apply them to examples.
    - Poisson random variable
    - Mean and variance of Poisson random variables

g. Moments and Moment-Generating Functions
  i. State the following theorems / define the terms, and apply them to examples.
    - kth moment of a random variable Y taken about the origin μ_k'
    - kth moment of a random variable Y taken about the mean
    - Moment generating function
    - If a moment generating function $m(t)$ exists, then $m^{(k)}(0) = \mu_k'$ for any positive integer k.

h. Tchebysheff's Theorem
  i. State the following theorems / define the terms, and apply them to examples.
    - Tchebysheff's Theorem

3. Continuous Variables and Their Probability Distributions

a. The Probability Distribution for a Continuous Random Variable
  i. State the following theorems / define the terms, and apply them to examples.
    - Cumulative distribution function
    - Properties of a distribution function
    - Continuous distribution function
    - Probability density function
    - Properties of a density function
    - Quantiles
    - If a random variable Y has density function f and $a < b$, then the probability that Y falls between a and b is the [Riemann] integral of f from a to b.

b. Expected Values for Continuous Random Variables
  i. State the following theorems / define the terms, and apply them to examples.
    - Expected value of a continuous random variable
    - Expected value of $g(Y)$ for a continuous random variable Y and a real-valued function g.
    - Variance and standard deviation (in terms of expected value)

        - Expected value of a constant
        - Linearity of expected value
    c. The Uniform Probability Distribution
        i. State the following theorems / define the terms, and apply them to examples.
            - Uniform probability distribution
            - Parameters of a density function
            - Mean and variance of a uniform random variable
    d. The Normal Probability Distribution
        i. State the following theorems / define the terms, and apply them to examples.
            - Normal probability distribution
            - Mean and variance of a normal random variable
    e. The Gamma Probability Distribution
        i. State the following theorems / define the terms, and apply them to examples.
            - Gamma distribution
            - Mean and variance of a random variable with gamma distribution
            - Chi-square distribution
            - Mean and variance of chi-square random variables
            - Exponential distribution
            - Mean and variance of exponential random variables

4. Multivariate Probability Distributions
    a. Bivariate and Multivariate Probability Distributions
        i. State the following theorems / define the terms, and apply them to examples.
            - Joint/bivariate probability function
            - For two discrete random variables Y_1, Y_2 with joint probability function $p(y_1, y_2)$, the function p is nonnegative and the sum of all $p(y_1, y_2)$ is 1.
            - Joint distribution function
            - Jointly continuous random variables
            - Joint probability density function
            - Properties of joint distribution functions
            - Properties of joint density functions
    b. Marginal and Conditional Distributions
        i. State the following theorems / define the terms, and apply them to examples.
            - Marginal probability functions
            - Marginal density functions
            - Conditional discrete probability function
            - Conditional distribution function
            - Conditional density
    c. Independent Random Variables
        i. State the following theorems / define the terms, and apply them to examples.
            - Independence and dependence with distribution functions
            - If Y_1, Y_2 are both discrete (or both continuous) random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions

$p_1(y_1)$, $p_2(y_2)$, then Y_1 and Y_2 are independent if and only if $p(y_1, y_2) = p_1(y_1)\, p_2(y_2)$ for all pairs of real numbers (y_1, y_2).

- The previous result holds for a joint density f where the support of f is a compact rectangle, and any nonnegative functions g and h may be used in place of p_1 and p_2, respectively.

d. The Expected Value of a Function of Random Variables
   i. State the following theorems / define the terms, and apply them to examples.
      - Expected value of a function of finitely many discrete [continuous] random variables

e. Special Theorems
   i. State the following theorems / define the terms, and apply them to examples.
      - Linearity of expected value for functions of two random variables
      - Multiplicative property of expected value for independent random variables

f. The Covariance of Two Random Variables
   i. State the following theorems / define the terms, and apply them to examples.
      - Covariance of two random variables
      - Properties of covariance
      - Independent random variables have zero correlation.

g. The Expected Value and Variance of Linear Functions of Random Variables
   i. State the following theorems / define the terms, and apply them to examples.
      - Properties of expected value, variance, and covariance on linear combinations of random variables

5. Sampling Distributions and the Central Limit Theorem
   a. The Central Limit Theorem
      i. State the following theorems / define the terms, and apply them to examples.
         - Central limit theorem for finitely many independently identically distributed random variables
   b. The Normal Approximation to the Binomial Distribution
      i. Given an example involving a binomial distribution, apply the normal approximation.