MTH 104
INTERMEDIATE ALGEBRA
FINAL EXAM REVIEW
DETAILED SOLUTIONS

4th edition
SPRING 2010

Solutions Prepared by
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Evaluate the function in problems: 1 – 3.

1. \( R(x) = -x^2 - 4x + 8 \); \( R(2) = (2)^2 - 4(2) + 8 = -4 + 8 = 4 \);

\( R(2) = -4 \)

2. \( f(x) = x^2 - 3x + 7 \); \( f(-2) = (-2)^2 - 3(-2) + 7 = 4 + 6 + 7 = 17 \);

\( f(-2) = 17 \)

3. \( g(x) = 2x^3 - 5x \); \( g(-2) = (2(-2))^3 - 5(-2) = -16 + 10 = -6 \);

\( g(-2) = -6 \)

Simplify problems: 4 – 12.
Assume all variables are positive.

4. \( \frac{2^{-5}x}{-3x^{-2}y^{-1}} = \frac{x^3y^4}{-3x^5} = \frac{x^4y^4}{-24} \)

5. \( (3a^{-2})^{-1}(3a^{-2})^3 = (3^{-1}a^2)(3^3a^{-3}) = 3^2a^{-1} = \frac{9}{a} \)

6. \( (27x^{12}y^{15})^{\frac{2}{3}} = (27^{\frac{2}{3}})(x^{12})^{\frac{2}{3}}(y^{15})^{\frac{2}{3}} = (\sqrt[3]{27})^2x^8y^{10} \)

\( = 9x^8y^{10} \)

7. \( \left(16y^{12}\right)^{\frac{3}{2}} = (16)^{\frac{3}{2}}(y^{12})^{\frac{3}{2}} = \left(\sqrt[2]{16}\right)^3(y^{12})^{\frac{3}{2}} = 8y^9 \)

8. \( \left(8x^9y^{12}z^{15}\right)^{\frac{2}{5}} = \left(8^2\right)^{\frac{2}{5}}\left(x^9\right)^{\frac{2}{5}}\left(y^{12}\right)^{\frac{2}{5}}\left(z^{15}\right)^{\frac{2}{5}} \)

\( = \left(\frac{2^2\left(x^3\right)^2\left(y^4\right)^2\left(z^5\right)^2}{5^2}\right)^{\frac{2}{5}} = \frac{4x^8y^8z^{10}}{25} \)

8a. Note: \( a^4 \) moved to the numerator becomes \( a^4 \); \( b^4 \) moved to the denominator becomes \( b^4 \); when the base is moved from the denominator to numerator or vice versa, the sign of the exponent changes to the opposite.

\( \frac{-3a^4b^3}{a^3b^7} = \frac{-3a^4}{b^4} \)

9. \( \left(\frac{1}{a^2b^{-1}}\right)^{\frac{2}{3}} = \frac{1}{a^2b^{-1}}\left(\frac{1}{a^2}\right)^{\frac{1}{3}} = \frac{1}{a^2b^{-1}}\frac{1}{a^2} = \frac{1}{a^4b^3} \)

Note about problem # 10: Divide the radical’s index 2 (index “2” is not marked over the square roots) into the exponents under the radical. The quotients resulting from that division become exponents over \( a \) and \( b \) (7 and 6, respectively) outside the radical and the remainder 1 becomes the non-written exponent over \( a \), under the radical. Problem # 11 is handled in a similar way.

10. \( \sqrt{75a^{16}b^{12}} = \sqrt{25\cdot3a^{16}b^{12}} = 5a^8b^6\sqrt{3a} \)

11. \( \sqrt[3]{-250x^5y^7} = \sqrt[3]{-125\cdot2x^3y^7} = -5xy\sqrt[3]{2x^3y} \)

12. \( \sqrt{-98} = i\sqrt{98} = i\sqrt{49\cdot2} = 7i\sqrt{2} \)

Factor completely problems: 13 – 19.

13. \( 15xz^2 + 6x - 5yz^2 - 2y = 3x(5z^2 + 2) - y(5z^2 + 2) = (3x - y)(5z^2 + 2) \) (Factored by grouping)

14. \( a^2 - 4ab + 4b^2 = (a - 2b)^2 \)

Perfect square trinomial \( \rightarrow \) Square of a binomial
15. \(5x^2 - 14x + 8 = (5x - 4)(x - 2)\)  
   - **Method:** Trial and Error  
   - **Solution:** \([5x - 4](x - 2)\)

16. \(100y^2 - 49 = (10y)^2 - (7)^2 = (10y + 7)(10y - 7)\)  
   - **Type:** Difference of two squares  
   - **Yield:** Product of the sum and difference of two terms

17. \(36a^2 + 42ab + 12b^2 = 6[6a^2 + 7ab + 2b^2]\)  
   - **Solution:** \(6(3a + 2b)(2a + b)\)

18. \(125a^3 + 8 = (5a)^3 + (2)^3\)  
   - **Solution:** \([(5a) + (2)][(5a)^2 - (5a)(2) + (2)^2]\)  
   - **Solution:** \((5a + 2)(25a^2 - 10a + 4)\)

19. \(64m^4 - 27mn^3 = m[(4m)^3 - (3n)^3]\)  
   - **Solution:** \(m(4m - 3n)(16m^2 + 12mn + 9n^2)\)

20. \((x+3)(x^2 - 2x + 1) = x^4 - 2x^2 + x + 3x^3 - 6x + 3\)  
   - **Solution:** \(x^4 + 3x^3 - 2x^2 - 5x + 3\)

21. \((3a + 2)^2 = (3a)^2 + 2(3a)(2) + (2)^2\)  
   - **Solution:** \(9a^2 + 12a + 4\)

22. \((4y - 3)(4y + 3) = (4y)^2 - (3)^2 = 16y^2 - 9\)  
   - **Type:** Product of the sum and difference of two terms  
   - **Yield:** Difference of two squares

23. \(\frac{3}{x+5} + \frac{24}{x^2 + 2x - 15} = \frac{3}{x+5} \cdot \frac{x - 3}{x^2 - 9} + \frac{24}{(x+5)(x-3)} = \frac{3x - 9}{(x+5)(x-3)} + \frac{24}{(x+5)(x-3)} = \frac{3(x+5)}{(x+5)(x-3)} = \frac{3}{x-3}\)

24. \(\frac{4}{x-2} - \frac{3x + 4}{x - 5} = \frac{4}{x-2} \cdot \frac{x - 5}{x - 5} - \frac{3x + 4}{x - 5} \cdot \frac{x - 2}{x - 2} = \frac{4x - 20}{(x-2)(x-5)} - \frac{3x^2 - 2x - 8}{(x-2)(x-5)} = \frac{4x - 20}{(x-2)(x-5)} - \frac{(3x^2 - 2x - 8)}{(x-2)(x-5)} = \frac{4x - 20 - 3x^2 + 2x + 8}{(x-2)(x-5)} = \frac{-3x^2 + 6x - 12}{(x-2)(x-5)} = \frac{-3(x^2 - 2x + 4)}{(x-2)(x-5)}\)

25. \(3\sqrt{3x^3} + 5x\sqrt{12x} = 3\sqrt{3x^3} + 5x\sqrt{4 \cdot 3x}\)  
   - **Solution:** \(3x\sqrt{3x} + 10x\sqrt{3x} = 13x\sqrt{3x}\)

26. \(5\sqrt{32} - \sqrt{72} = 5\sqrt{16 \cdot 2} - \sqrt{36 \cdot 2}\)  
   - **Solution:** \(5 \cdot 4\sqrt{2} - 6\sqrt{2} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}\)
### 27
\[
\sqrt[3]{40x} - \sqrt[3]{5x} = \frac{3}{8} \sqrt[3]{5x} - \sqrt[3]{5x} = 2\sqrt[3]{5x} - \sqrt[3]{5x} = \frac{5x}{\sqrt[3]{5x}}
\]

### 28
\[
\frac{x^2 + 3x - 18}{12 - x - x^2} = \frac{x^2 + 2x - 24}{x^2 + 10x + 24} = \frac{(x + 6)(x - 3)}{(-x + 3)(x + 4)} \cdot \frac{(x + 6)(x - 4)}{(x + 6)(x - 4)} = \frac{x + 6}{x - 4}
\]


#### 29
\[
(x^2 - 12x + 32) ÷ (x - 4)
\]

\[
x - 8
\]

\[
x^2 - 4x
\]

\[
-8x + 32
\]

\[
-8x + 32
\]

\[
R = 0
\]

**Ans.: x – 8**

#### 30
\[
x^2 + 2x - 5x - 6
\]

\[
x^2 + 4x + 3
\]

\[
x - 2
\]

\[
x^2 + 2x - 5x - 6
\]

\[
x^2 - 2x
\]

\[
4x^2 - 5x
\]

\[
4x^2 - 8x
\]

\[
R = 0
\]

**Ans.: x^2 + 4x + 3**

#### 31
\[
(12x^2 + 10x - 5) ÷ (2x + 1)
\]

\[
6x + 2
\]

\[
12x^2 + 10x - 5
\]

\[
2x + 1
\]

\[
4x - 5
\]

\[
4x + 2
\]

\[
R = -7
\]

**Ans.: 6x + 2 - \frac{7}{2x + 1}**

#### 32
\[
8 \div \sqrt{x - 2} = \frac{8}{\sqrt{x - 2}} \cdot \frac{\sqrt{x + 2}}{\sqrt{x + 2}}
\]

\[
\frac{8}{\sqrt{x} + 2}
\]

\[
8(\sqrt{x} + 2)
\]

\[
\frac{8}{(\sqrt{x})^2 - (2)^2}
\]

\[
\frac{8\sqrt{x} + 16}{x - 4}
\]

#### 33
\[
\frac{9}{y^2 + \frac{3}{y}} = \frac{9}{y^2 + \frac{3}{y}} \cdot \frac{y^2}{y^2 + 3y} = \frac{9y^2}{y(y^2 + 3y)} = \frac{3y}{y}
\]

#### 34
\[
\frac{2}{y} - \frac{4}{y^3} = \frac{2}{y} - \frac{4}{y^3} \cdot \frac{y(y - 3)}{y(y - 3)} = \frac{2(y - 3) - 4y}{y(y - 3)} = \frac{2y - 6 - 4y}{y(y - 3)} = \frac{-2y - 6}{y^3 - 4y - 3}
\]

\[
\frac{1}{y} - \frac{3}{y - 3} = \frac{1}{y} - \frac{3}{y - 3} \cdot \frac{y(y - 3)}{y(y - 3)} = \frac{y(y - 3) - 3y}{y(y - 3)} = \frac{y - 3y}{y^3} = \frac{-2y}{y^3 - 4y - 3}
\]
### Solve the equation. For the specified variable: 35 – 37.

<table>
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<th>Question Number</th>
<th>Equation</th>
<th>Solution</th>
<th>Notes</th>
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<tr>
<td><strong>35</strong></td>
<td>( a = ? ) [ v = at + v_0 \rightarrow v - v_0 = at + y' - y' ]</td>
<td>[ a = \frac{v - v_0}{t} ]</td>
<td></td>
</tr>
<tr>
<td><strong>36</strong></td>
<td>[ mv + mp = bv \rightarrow mv + mp - mp = bv - mp ]</td>
<td>[ v = ? ] [ mv = bv - mp \rightarrow mv - bv = -mp ]</td>
<td>[ v(m - b) = -mp \Rightarrow v = -\frac{mp}{m - b} = \frac{mp}{b - m} ]</td>
</tr>
<tr>
<td><strong>37</strong></td>
<td>[ r = ? ] [ S = \frac{a}{1 - r} \rightarrow \left( S = \frac{a}{1 - r} \right)(1 - r) \rightarrow (1 - r)S = a \rightarrow S - S_r = a \rightarrow S_r = S - a ]</td>
<td>[ r = \frac{S - a}{S} = -\frac{a - S}{S} ]</td>
<td></td>
</tr>
</tbody>
</table>

### Solve the equation. Include any complex solutions: 38 – 43.

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<tr>
<th>Question Number</th>
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<th>Notes</th>
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<tr>
<td><strong>38</strong></td>
<td>[ \sqrt{5x + 1} = 4 \rightarrow (\sqrt{5x + 1})^2 = (4)^2 \rightarrow 5x + 1 = 16 ]</td>
<td>[ 5x = 15 \rightarrow x = 3 ] Must check: ( \sqrt{5(3) + 1} = 4 )? [ \sqrt{16} = 4 \Rightarrow 4 = 4 ]</td>
<td>[ \therefore \text{Solution set} = {3} ]</td>
</tr>
<tr>
<td><strong>39</strong></td>
<td>[ 5 + \sqrt{4x + 8} = 11 \rightarrow \sqrt{4x + 8} = 6 ]</td>
<td>[ \left( \sqrt{4x + 8} \right)^2 = (6)^2 \rightarrow 4x + 8 = 36 ] [ 4x = 28 \rightarrow x = 7 ] Must check: [ 5 + \sqrt{4(7) + 8} = 11 \text{?} ] [ 5 + \sqrt{36} = 11 \rightarrow 5 + 6 = 11 ] [ 11 = 11 \therefore \text{Solution set} = {7} ]</td>
<td></td>
</tr>
<tr>
<td><strong>40</strong></td>
<td>Multiply both sides of the equation by LCD to eliminate the denominators. [ \frac{3}{x^2 - 25} + \frac{1}{x + 5} = \frac{8}{x - 5} \rightarrow \frac{3}{(x + 5)(x - 5)} + \frac{1}{x + 5} = \frac{8}{x - 5} ] LCD = ((x + 5)(x - 5))</td>
<td>[ \left( \frac{3}{(x + 5)(x - 5)} + \frac{1}{x + 5} \right) \cdot (x + 5)(x - 5) \rightarrow 3 + (x - 5) = 8(x + 5) \rightarrow -2 + x = 8x + 40 ] [ 7x = -42 \rightarrow x = -\frac{42}{7} \rightarrow x = -6; \ D/C \text{ (Denominator check): (-6)+5 \neq 0, (-6)-5 \neq 0}; ]</td>
<td>[ \therefore \text{Solution set} = {-6} ]</td>
</tr>
<tr>
<td><strong>41</strong></td>
<td>Multiply both sides of the equation by LCD to eliminate the denominators. [ \left( \frac{-6}{a - 6} = 4 - \frac{a}{a - 6} \right) \cdot (a - 6) \rightarrow -6 = 4(a - 6) - a \rightarrow -6 = 4a - 24 - a \rightarrow -6 = 3a - 24 \rightarrow 3a = 18 ]</td>
<td>[ a = \frac{18}{3} \rightarrow a = 6 ] D/C (Denominator check): ( 6 - 6 = 0 \therefore \text{Solution set} = {} = \emptyset ]</td>
<td>Since the denominator ( = 0 ), and since division by zero yields undefined result, there is no solution to the equation.</td>
</tr>
</tbody>
</table>
Solve each inequality, express the solution using set notation, interval notation, and graph the solution on the real number line (44 – 52).

44. 
-2 < 3x + 7 < 4
\[-7 \leq 3x \leq -4\]
\[-9 < 3x < -3\] \div (3)

Set notation:
\{ x \mid -3 < x < -1 \}

Interval notation:
\(-3, -1\)

45. 
-3 < x < -1

“AND” Inequality
Tip is in, expression in between
\(|x - 3| \leq 5 \rightarrow -5 \leq x - 3 \leq 5 \rightarrow -2 \leq x \leq 8\)

Interval notation:
\([-2, 8]\)

46. 
“OR” Inequality
Tip is out, expression left and right.
\(|5 - x| \geq 2 \rightarrow 5 - x \leq -2 \text{ or } 5 - x \geq 2\)

Multiply by -1 and reverse the inequalities.
\[x \geq 7 \text{ or } x \leq 3\]

Interval notation:
\((-\infty, 3] \cup [7, \infty)\)

47. 
(a) \(x \geq 1\) OR \(x > 7\)

(b) \(x = 7\)

Solution:
\(x \geq 1\)

Set notation:
\{ x \mid x \geq 1 \}

Interval notation:
\([1, \infty)\)

48. 
(a) \(x < 3\) AND \(x \leq 7\)

(b) \(x = 3\)

Solution:
\(x < 3\)

Set notation:
\{ x \mid x < 3 \}

Interval notation:
\((-\infty, 3]\)

49. 
(a) \(x \geq 4\) OR \(x < 7\)

(b) \(x = 4\)

Solution:
All real numbers

Set notation:
\{ x \mid x \text{ is a real number} \}

Interval notation:
\((-\infty, \infty)\)
50. \( x < 3 \) AND \( x > 10 \)

Solution: No solution

Set notation: \{ \} = \emptyset

51. \( x < 0 \) OR \( x \geq 6 \)

Solution:

Set notation: \{ x | x < 0 or x \geq 6 \}

Interval notation: \(( -\infty, 0 \) \cup \( [6, \infty) \)

52. \( x \leq 0 \) AND \( x > -2 \)

Solution:

Set notation: \{ x | -2 < x \leq 0 \}

Interval notation: \([-2, 0]\)

53. Solve the quadratic equation by completing the square. Express the solution in set notation: (53 – 54).

\[
\begin{align*}
x^2 - 6x - 16 &= 0 \\
x^2 - 6x + \left(\frac{6}{2}\right)^2 &= 16 + \left(\frac{6}{2}\right)^2 \\
x^2 - 6x + (3)^2 &= 16 + 3^2 \\
(x-3)^2 &= 16 + 9 \\
(x-3)^2 &= 25 \\
(x-3) &= \pm 5 \\
x &= -2 \text{ or } x = 8 \\
\text{Solution set} &= \{-2, 8\}
\end{align*}
\]

54. \[
\begin{align*}
y^2 + 10y + 22 &= 0 \\
y^2 + 10y + \left(\frac{10}{2}\right)^2 &= -22 + \left(\frac{10}{2}\right)^2 \\
y^2 + 10y + (5)^2 &= -22 + (5)^2 \\
(y + 5)^2 &= -22 + 25 \\
(y + 5)^2 &= 3 \\
\sqrt{(y + 5)} &= \pm \sqrt{3} \\
y + 5 &= \pm \sqrt{3} \\
y &= -5 - \sqrt{3} \text{ or } y = -5 + \sqrt{3}
\end{align*}
\]

Solution set = \{-5 - \sqrt{3}, -5 + \sqrt{3}\}
Solve the quadratic equation by using the quadratic formula. Express the solution in set notation. (55 – 57).

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<tr>
<th>Problem</th>
<th>Quadratic Formula</th>
<th>Solution</th>
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<tr>
<td>55</td>
<td>( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
<td>( x = \frac{-8 \pm \sqrt{64 + 48}}{8} = \frac{-8 \pm \sqrt{112}}{8} = \frac{-8 \pm \sqrt{16 \times 7}}{8} = -2 \pm \frac{\sqrt{7}}{2} ) ( \implies ) Solution set = ( \left{ \frac{-2 - \sqrt{7}}{2}, \frac{-2 + \sqrt{7}}{2} \right} )</td>
</tr>
<tr>
<td>56</td>
<td>( 20a - 25 = 4a^2 \rightarrow 4a^2 - 20a + 25 = 0 )</td>
<td>( a = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} = \frac{20 \pm \sqrt{400 - 400}}{8} = \frac{20 \pm 0}{8} = \frac{20}{8} = \frac{5}{2} ) ( \implies ) Solution set = ( \left{ \frac{5}{2} \right} )</td>
</tr>
<tr>
<td>57</td>
<td>( x^2 + 2x + 2 = 0 )</td>
<td>( x = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm i\sqrt{4}}{2} = -1 \pm i ) ( \implies ) Solution set = ( \left{ -1 - i, -1 + i \right} )</td>
</tr>
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Use the discriminant to determine whether the quadratic equation has one real number solution, two real number solutions, or two complex number solutions: (58 – 59).

<table>
<thead>
<tr>
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<th>Discriminant</th>
<th>Quadratic Formula</th>
<th>Solution</th>
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<tr>
<td>58</td>
<td>( 9x^2 + 30x + 25 = 0 ) ( \implies ) ( b^2 - 4ac = (30)^2 - 4(9)(25) = 900 - 900 = 0 )</td>
<td>( \therefore ) The Quadratic Formula will yield only one real solution.</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>( 3t^2 - t + 2 = 0 ) ( \implies ) ( b^2 - 4ac = (-1)^2 - 4(3)(2) = 1 - 24 = -23 )</td>
<td>( \therefore ) The Quadratic Formula will yield two complex solutions.</td>
<td></td>
</tr>
</tbody>
</table>
Perform the indicated operations and simplify. Write the result in \( a + bi \) form:

\((60 - 63)\).

\[
5i (3 - 2i) = 15i - 10i^2 = 15i + 10 = \boxed{10 + 15i}
\]

\[
(3+2i)^2 = (3)^2 + 2(3)(2i) + (2i)^2 = 9 + 12i + 4i^2 = 9 + 12i - 4 = \boxed{5 + 12i}
\]

\[
\frac{3+2i}{7-6i} = \frac{3+2i \cdot 7+6i}{7-6i \cdot 7+6i} = \frac{21+18i+14i+12i^2}{(7)^2 - (6i)^2} = \frac{21+32i-12}{49-36i^2} = \frac{9+32i}{49+36} = \frac{9+32i}{85} = \boxed{\frac{9}{85} + \frac{32}{85}i}
\]

\[
\frac{4}{3-7i} = \frac{4 \cdot 3+7i}{(3-7i)(3+7i)} = \frac{12+28i}{3^2 -(7i)^2} = \frac{12+28i}{9+49} = \frac{12+28i}{58} = \boxed{\frac{6}{29} + \frac{14}{29}i}
\]

64. Use a calculator to evaluate to four decimal places: \( \cos 80° \approx 0.1736 \)

65. Use a calculator to evaluate to four decimal places: \( \tan 14.1° \approx 0.2512 \)

Solve using right triangle ABC with \( \angle C = 90° \): problems 66 – 70.

Use exact values in your answers (leave radicals, no decimals), unless otherwise indicated.

\( c = 3 \text{ cm}, \ b = 2 \text{ cm}, \ a = ? \)

(i) Find the exact value of side \( a \)

\[ c^2 = a^2 + b^2 \rightarrow a^2 = c^2 - b^2 \rightarrow a = \sqrt{c^2 - b^2} \rightarrow a = \sqrt{3^2 - 2^2} \rightarrow a = \sqrt{5} \]

(ii) Find the exact value of: \( \sin A, \cos A, \tan A \)

\[ \sin A = \frac{a}{c} \rightarrow \sin A = \frac{\sqrt{5}}{3} \] \[ \cos A = \frac{b}{c} \rightarrow \cos A = \frac{2}{3} \] \[ \tan A = \frac{a}{b} \rightarrow \tan A = \frac{\sqrt{5}}{2} \]

(iii) Find, to the nearest tenth: \( \angle A \) and \( \angle B \)

From calculator: \( \angle A = \cos^{-1} \left( \frac{2}{3} \right) = 48.2° \) OR \( \angle A = \sin^{-1} \left( \frac{\sqrt{5}}{3} \right) = 48.2° \)

From the triangle: since \( \angle A + \angle B = 90° \rightarrow \angle B = 90° - \angle A \rightarrow \angle B = 90° - 48.2° = 41.8° \)

\( \therefore \ \angle A \approx 48.2° \) and \( \angle B = 41.8° \)
67. \( \tan A = \frac{a}{b} \rightarrow a = b \cdot \tan A \rightarrow a = 5 \cdot \tan 28^\circ \rightarrow a \approx 5 \cdot (0.5317) \rightarrow a \approx 2.7 \)
\[
\cos A = \frac{b}{c} \rightarrow c = \frac{b}{\cos A} \rightarrow c = \frac{5}{\cos 28^\circ} \rightarrow c = \frac{5}{0.8829} \rightarrow c \approx 5.7
\]

68. \( a = 2, \quad b = 5; \quad \text{Find:} \quad \sin A = ? \quad \cos A = ? \quad \tan A = ? \quad (\text{exact values, no decimals}) \)
\[
c^2 = a^2 + b^2 \rightarrow c^2 = (2)^2 + (5)^2 \rightarrow c^2 = 4 + 25 \rightarrow c^2 = 29 \rightarrow c = \sqrt{29}
\]
\[
\sin A = \frac{a}{c} = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2 \sqrt{29}}{29} \quad \therefore \sin A = \frac{2 \sqrt{29}}{29}
\]
\[
\cos A = \frac{b}{c} = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5 \sqrt{29}}{29} \quad \therefore \cos A = \frac{5 \sqrt{29}}{29}
\]
\[
\tan A = \frac{a}{b} = \frac{2}{5} \quad \therefore \tan A = \frac{2}{5}
\]

69. Hypotenuse \( c = 11 \), one leg: \( a = 7 \); Find the three trig functions of the smaller angle.
\[
\sin A = \frac{a}{c} = \frac{7}{11} = 0.6364 \quad \therefore \sin A \approx 0.6364
\]
\[
\cos A = \frac{b}{c} = \frac{6 \sqrt{2}}{11} = 0.7714 \quad \therefore \cos A \approx 0.7714
\]
\[
\tan A = \frac{a}{b} = \frac{7}{6 \sqrt{2}} = 0.8250 \quad \therefore \tan A = 0.8250
\]

70. \( \alpha B = 42^\circ, \quad a = 9; \quad \text{Find all other angles and sides; round to one decimal place:} \)
\[
A = 90^\circ - 42^\circ = 48^\circ \quad \text{Finding side} \ c
\]
\[
\sin A = \cos B = \frac{a}{c} \rightarrow \sin 48^\circ = \frac{9}{c} \rightarrow c = \frac{9}{\sin 48^\circ} = \frac{9}{0.7431} = 12.1
\]
\[
\tan B = \frac{b}{a} \rightarrow \tan 42^\circ = \frac{b}{9} \rightarrow b = 9 \cdot \tan 42^\circ = 9 \cdot (0.9004) = 8.1
\]

**ANSWERS:** \( A = 48^\circ, \quad b = 8.1, \quad c \approx 12.1 \)
### Find the equation of the line that satisfies the information: problems 71 – 73.

Write your answers in slope-intercept form.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation of the line</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>71</strong></td>
<td>Equation of the line through the point (4,-1), with a slope of (\frac{3}{4}); in slope-intercept form.</td>
<td>(y - y_i = m(x - x_i) \rightarrow y - (-1) = \frac{3}{4}(x - 4) \rightarrow y + 1 = \frac{3}{4}x - 3 \rightarrow y = \frac{3}{4}x - 4)</td>
</tr>
<tr>
<td><strong>72</strong></td>
<td>Equation of the line through the points (-2, 2) and (1, 3); in slope-intercept form.</td>
<td>(m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{3 - 2}{1 - (-2)} \rightarrow m = \frac{1}{3} \rightarrow m = 1 \rightarrow y - y_i = m(x - x_i) \rightarrow y - 3 = \frac{1}{3}(x - 1) \rightarrow y - 3 = \frac{1}{3}x - \frac{1}{3} \rightarrow y = \frac{1}{3}x + \frac{8}{3})</td>
</tr>
<tr>
<td><strong>73</strong></td>
<td><strong>(a)</strong> Equation of the line through the point (3, 1) and perpendicular to the line (2x + y = -3); in slope-intercept form. (l_1) (2x + y = -3 \rightarrow y = -2x - 3 \rightarrow m_1 = -2) (l_2) (m_2 = \frac{-1}{m_1} \rightarrow m_2 = \frac{-1}{-2} \rightarrow m_2 = \frac{1}{2})</td>
<td>(y - y_i = m(x - x_i) \rightarrow y - 1 = \frac{1}{2}(x - 3) \rightarrow y - 1 = \frac{1}{2}x - \frac{3}{2} \rightarrow y = \frac{1}{2}x - \frac{1}{2})</td>
</tr>
<tr>
<td><strong>73</strong></td>
<td><strong>(b)</strong> Equation of the line through the point (3, 1) and parallel to the line (2x + y = -3); in slope-intercept form. (l_1) (2x + y = -3 \rightarrow y = -2x - 3 \rightarrow m_1 = -2) (l_2) (m_2 = m_1 \rightarrow m_2 = -2)</td>
<td>(y - y_i = m(x - x_i) \rightarrow y - 1 = -2(x - 3) \rightarrow y - 1 = -2x + 6 \rightarrow y = -2x + 6 + 1 \rightarrow y = -2x + 7)</td>
</tr>
</tbody>
</table>
74. Graph the solution to the inequality $x + 2y > 6$

FOUR STEPS
1. Write the corresponding equation, in order to draw the boundary line: $x + 2y = 6$; find the intercepts (0,3) and (6,0) and plot the points.
2. Draw a dashed/dotted boundary line.
3. Check if the point (0,0) satisfies the inequality: 
   $(0) + 2(0) > 6$? It does not.
4. Hence, shade the solution to the inequality on the other side of the boundary line than the point (0,0) is found.

75. Graph the solution to the inequality $3x - 5y < 15$

FOUR STEPS
1. Write the corresponding equation, in order to draw the boundary line: $3x - 5y = 15$; find the intercepts (0,-3) and (5,0) and plot the points.
2. Draw a dashed/dotted boundary line.
3. Check if the point (0,0) satisfies the inequality: 
   $3(0) - 5(0) < 15$? It does.
4. Hence, shade the solution to the inequality on the same side of the boundary line that the point (0,0) is found.

76. Given the equation $y = x^2 + 6x + 5$, do the following:

<table>
<thead>
<tr>
<th></th>
<th>Find the vertex and the equation of the axis of symmetry</th>
<th>Find the vertex and the equation of the axis of symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Graph the quadratic equation on the coordinate axes below and label the points with the coordinates.</td>
<td>Graph the quadratic equation on the coordinate axes below and label the points with the coordinates.</td>
</tr>
<tr>
<td>2</td>
<td>State the y-intercept.</td>
<td>State the y-intercept.</td>
</tr>
<tr>
<td>3</td>
<td>State the x-intercepts, if any.</td>
<td>State the x-intercepts, if any.</td>
</tr>
<tr>
<td>4</td>
<td>NOTE: Since $a &gt; 0$, parabola opens up.</td>
<td>NOTE: Since $a &gt; 0$, parabola opens up.</td>
</tr>
</tbody>
</table>

SOLUTION:

1. VERTEX (VTX) and the Equation of Symmetry

When the parabola opens upward, the vertex is the lowest point on the curve.
Find the vertex coordinates:

\[ x = \frac{-b}{2a} = \frac{-6}{2(1)} = -3; \quad \boxed{x = -3} \]

is both the x-coordinate of the VERTEX and the equation of the axis of symmetry of the parabola. To find the y-coordinate, substitute \( x = -3 \) in the equation:

\[ y(-3) = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4 \]

Hence, the VTX coordinates are: \((-3, -4)\).

2. **Y-INTERCEPT and the Y-INTERCEPT’S “COUSIN POINT”**

To find the y-intercept, it is necessary to set \( x = 0 \) in the equation.

\[ y = x^2 + 6x + 5 \]

\[ y(0) = (0)^2 + 6(0) + 5 = 5 \]

Hence, the y-intercept is \((0, 5)\).

On the other side of the axis of symmetry, equally distant from that axis is the “cousin point” whose coordinates are \((-6, 5)\); this may also be determined by inspection.

3. **X-INTERCEPTS**

To find the x-intercepts, it is necessary to set \( y = 0 \) in the equation.

\[ 0 = x^2 + 6x + 5 \]

\[ (x + 1)(x + 5) = 0 \]

So \( x = -1 \) and \( x = -5 \)

Hence, the x-intercepts are: \((-1, 0)\) and \((-5, 0)\).

**IMPORTANT:** X-intercepts should only be considered if the equation is easy to factor. Otherwise, plotting additional points, as shown in step 4 will suffice.

4. **PLOTTING ADDITIONAL POINTS**

To plot additional points, it is convenient to use a table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-4</td>
<td>-3</td>
</tr>
</tbody>
</table>

Hence, the additional points are: \((-2, -3)\) and \((-4, -3)\).

These points are also symmetrically located (“cousins”) and when we know one, we can, by inspection, determine the other; as it was done in Step 2. This completes the graph.

**ANSWERS:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vertex: ((-3, -4)); axis of symm.: (x = -3)</td>
</tr>
<tr>
<td>2</td>
<td>Y-intercept: ((0, 5))</td>
</tr>
<tr>
<td>3</td>
<td>X-intercepts: ((-1, 0)) and ((-5, 0))</td>
</tr>
<tr>
<td>4</td>
<td>The graph is shown above, in Step #4.</td>
</tr>
</tbody>
</table>
77. Given the equation \( y = -x^2 + 4x - 3 \), do the following:

<table>
<thead>
<tr>
<th></th>
<th>Find the vertex and the equation of the axis of symmetry</th>
<th>Graph the quadratic equation on the coordinate axes below and label the points with the coordinates.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>State the y-intercept.</td>
<td>NOTE: Since ( a &lt; 0 ), parabola opens downward.</td>
</tr>
<tr>
<td>2</td>
<td>State the x-intercepts, if any.</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**

1. **VERTEX (VTX) and the Equation of Symmetry**
   
   When the parabola opens downward, the vertex is the highest point on the curve.
   
   **Find the vertex coordinates:**
   
   \[
   x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2; \quad [x = 2] \text{ is both the x-coordinate of the VTX and the equation of the axis of symmetry of the parabola.}
   \]
   
   To find the y-coordinate, substitute \( x = 2 \) in the equation:
   
   \[
   y (2) = -(2)^2 + 4(2) - 3 = -1
   \]
   
   Hence, the VTX coordinates are: \( (2, 1) \).

2. **Y-INTERCEPT and the Y-INTERCEPT'S “Cousin Point”**
   
   To find the y-intercept, it is necessary to set \( x = 0 \) in the equation.
   
   \[
   y = -x^2 + 4x - 3 \Rightarrow y (0) = -(0)^2 + 4(0) - 3 = -3
   \]
   
   Hence, the y-intercept is \( (0, -3) \).
   
   On the other side of the axis of symmetry, equally distanced from that axis is the "cousin point" whose coordinates are \( (4, -3) \); this may also be determined by inspection.

3. **X-INTERCEPTS**
   
   To find the x-intercepts, it is necessary to set \( y = 0 \) in the equation.
   
   \[
   0 = y = -x^2 + 4x - 3 \Rightarrow (-x + 1)(x - 3) = 0 \quad \text{So} \quad x = 1, \quad x = 3
   \]
   
   Hence, the x-intercepts are: \( (1, 0) \) and \( (3, 0) \).

**IMPORTANT:** X-intercepts should only be considered if the equation is easy to factor. Otherwise, plotting additional points, as shown in step 4, will suffice.
4. PLOTTING ADDITIONAL POINTS

To plot additional points, it is convenient to use a table:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>5</td>
<td>-8</td>
</tr>
</tbody>
</table>

Hence, the additional points are: (-1, -8) and (5, -8).

These points are also symmetrically located ("cousins") and when we know one, we can, by inspection, determine the other; as it was done in Step 2. This completes the graph.

ANSWERS:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vertex: (2, 1); axis of symm.: x = 2</td>
<td>3</td>
<td>X-intercepts: (1, 0) and (3, 0)</td>
</tr>
<tr>
<td>2</td>
<td>Y-intercept: (0, -3)</td>
<td>4</td>
<td>The graph is shown above, in Step #4.</td>
</tr>
</tbody>
</table>

78. Solve the system of equations by graphing.

(a) \( y = 2x + 1 \)
(b) \( x + y = -2 \)

Using the equation (a), obtain two ordered pairs, plot the two points and draw a line.

Using the equation (b), obtain two ordered pairs, plot the two points and draw a line.

Solution is where the two lines intersect, at (-1, -1). The system is consistent.
Solve the system of equations by graphing.

79.  

(a) \( x + 2y = 6 \)  
(b) \( y = -\left(\frac{1}{2}\right)x + 3 \)

The two lines intersect on their entire length, because they are superposed. Hence, there are an infinite number of solutions. The system is dependent.

---

### Solve the system of equations algebraically: problems 80 – 83.

(Use either substitution or addition/elimination method).

<table>
<thead>
<tr>
<th>80</th>
<th>Addition/elimination method</th>
<th>Substitution method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (2x + 3y = 5) (\text{SOLUTION}) (\rightarrow y = 1) (\text{Sol. set} = {(1,1)})</td>
<td>(a) (2x + 3y = 5) (\text{SOLUTION}) (\rightarrow y = 1) (\text{Sol. set} = {(1,1)})</td>
<td></td>
</tr>
<tr>
<td>(b) (x + 2y = 3) (\text{SOLUTION}) (\rightarrow y = 1) (\text{Sol. set} = {(1,1)})</td>
<td>(b) (x + 2y = 3) (\text{SOLUTION}) (\rightarrow y = 1) (\text{Sol. set} = {(1,1)})</td>
<td></td>
</tr>
<tr>
<td>(-2b) (\Rightarrow 2x - 4y = -6) (\rightarrow y = -1) (\Rightarrow \text{Sol. set} = {(1,1)})</td>
<td>(-2b) (\Rightarrow x = 3 - 2y) (\Rightarrow \text{SOLUTION}) (\rightarrow y = 1) (\Rightarrow \text{Sol. set} = {(1,1)})</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>81</th>
<th>Addition/elimination method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (2x - 6y = 5) (\text{SOLUTION}) (\rightarrow 0 \neq -5) No solution.</td>
<td>(b) (4x - 12y = 5) (\text{SOLUTION}) (\rightarrow 0 \neq -5) No solution.</td>
</tr>
<tr>
<td>((\Rightarrow 2a)) (\Rightarrow 4x + 12y = -10) Contradiction, the system is inconsistent.</td>
<td>((\Rightarrow b)) (\Rightarrow 4x - 12y = 5) The lines are parallel, hence do not intersect.</td>
</tr>
</tbody>
</table>

NOTE: Another way to show that the system is inconsistent (equations represent parallel lines: no intersection point), would be to write them in slope-intercept form: same slopes, different \(b\)'s.
(i) 
(a) $x + y - z = -2$
(b) $2x - y + z = -1$
(c) $x + 2y - 3z = -7$

(ii) 
(a) $x + y + z = 2$
(b) $2x + y - z = -1$

(a + b) $3x = -3$

$x = -1$

(iii) 
(a) $-1 + y - z = 2$
(b) $-1 + 2y - 3z = -7$

Simplify, rename (A, C):
(A) $y - z = -1$
(C) $2y - 3z = -6$

(iv) 
Solve for: $z$

(-2A) $2y + 2z = 2$

(C) $2y - 3z = 6$

$z = 4$

(v) 
Substitute to (a) and solve for: $y$

(a) $-1 + y - 4 = -2$

$y - 5 = -2$

$y = 3$

Solution set = \{(-1, 3, 4)\}

Check if the solution satisfies all three equations.

(iv) 
Solve for: $x$

(-1•A) $-3x + 2y = 6$

(B) $8x - z = 11$

We have now one equation with two variables, $x & z$; another is needed.

(iii) 

(-2b) $-10x + 6y - 10z = -4$

(3c) $18x - 6y + 9z = 15$

(B) $8x - z = 11$

Superpose equations (A, B):
(A) $3x - z = 6$
(B) $8x - z = 11$

(v) 
Substitute $x = 1$ to (A) and solve for: $z$

(A) $3(1) - z = 6$

$3 - z = 6$

$z = -3$

(v) 
Substitute $x = 1$ & $z = -3$ to (a), (b) or (c); find $y$

(a) $3(1) - 2y + 4(-3) = -1$

$-2y = 8$

$y = -4$

Solution set = \{(1, -4, -3)\}

Check if the solution satisfies all three equations.
For the following word problems, identify the variable(s) used, set up equation(s) and solve algebraically.

84. A 30-foot ladder, leaning against the side of a building, makes a 50° angle with the ground. How far up the building does the top of the ladder reach? Express your answer to the nearest tenth of a foot.

**VARIABLE(S):**
- \( x \) = distance from the top of the ladder to the base of the house.
- \( L \) = 30 ft. (Length of the ladder)

\[
\sin 50^\circ = \frac{x}{L} \quad \rightarrow \quad x = L \cdot \sin 50^\circ
\]
\[
x = (30) \cdot (0.7660) = 22.98 \approx 23.0\text{ ft.} \quad \boxed{x = 23\text{ ft.}}
\]

**ANSWER:**
The top of the ladder reaches 23.0 feet up the side of the building.

85. A 70 foot rope is attached to the top of one of the vertical poles used to hold up a circus tent. The other end of the rope is anchored to the ground 40 feet from the bottom of the pole. What is the height of the tent and what angle does the rope make with the tent? Express both answers to the nearest tenth (remember about the units).

**VARIABLE(S):**
- \( \Theta \) = angle of the rope with the tent
- \( h \) = height of the tent

\[
\sin \theta = \frac{40}{70} \quad \rightarrow \quad \sin \theta = \frac{4}{7} \quad \rightarrow \quad \sin^{-1} \left( \frac{4}{7} \right) \approx 34.8^\circ
\]

**Pythagorean Theorem:**
\[
a^2 + b^2 = c^2 \quad \rightarrow \quad 40^2 + h^2 = 70^2 \quad \rightarrow \\
h^2 = 70^2 - 40^2 \quad \rightarrow \quad h = \sqrt{4900 - 1600} \quad \rightarrow \quad h = \sqrt{3300}
\]

**ANSWERS:** The height of the tent is 57.4 ft. The angle \( \Theta \approx 34.8^\circ \)

86. Find three consecutive odd integers such that seven times the sum of the first two integers is three more than nine times the third integer.

**VARIABLES:**
- \( x \) = the first odd integer;
- \( x + 2 \) = the second odd integer;
- \( x + 4 \) = the third odd integer

\[
7 \cdot (x + (x + 2)) = 9 \cdot (x + 4) + 3 \quad \rightarrow \quad 7 \cdot (2x + 2) = 9x + 36 + 3 \quad \rightarrow \\
14x + 14 = 9x + 39 \quad \rightarrow \quad 5x = 25 \quad \rightarrow \quad x = 5
\]

**ANSWER:** The integers are: 5, 7 and 9
87. A chemist must mix 8 L of a 40% acid solution with some 70% acid solution to get 50% acid solution. How much of the 70% solution should be used?

**ID – RAG**

<table>
<thead>
<tr>
<th>Solution Type</th>
<th>Acid % Decimals</th>
<th>Solution liters</th>
<th>Pure acid liters</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% solution</td>
<td>0.40</td>
<td>8</td>
<td>(0.40) (8) = 3.2</td>
</tr>
<tr>
<td>70% solution</td>
<td>0.70</td>
<td>x</td>
<td>0.70 x</td>
</tr>
<tr>
<td>50% solution</td>
<td>0.50</td>
<td>(8 + x)</td>
<td>0.50 (8 + x)</td>
</tr>
</tbody>
</table>

**THE AMOUNT OF PURE ACID BEFORE MIXING = THE AMOUNT OF PURE ACID AFTER MIXING**

\[(0.40) (8) + 0.70 x = 0.50 (8 + x) \rightarrow 3.20 + 0.70 x = 4.00 + 0.50 x \rightarrow 0.20 x = 0.80 \]

\[x = \frac{0.80}{0.20} = 4\]

\[\rightarrow x = 4\]

**ANSWER:**

Needed are 4 liters of 70% acid solution.

88. A private airplane leaves an airport and flies due east at 180 mph. Two hours later, a jet leaves the same airport and flies due east at 900 mph. How long will it take for the jet to overtake the private plane?

**ID – RAG**

<table>
<thead>
<tr>
<th>Type of Plane</th>
<th>Speed mph</th>
<th>Time hrs</th>
<th>Distance mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Airplane</td>
<td>180</td>
<td>t + 2</td>
<td>180 (t + 2)</td>
</tr>
<tr>
<td>The Jet</td>
<td>900</td>
<td>t</td>
<td>900 t</td>
</tr>
</tbody>
</table>

**DISTANCE TRAVELED BY PRIVATE AIRPLANE = DISTANCE TRAVELED BY JET**

\[180 (t + 2) = 900 t \rightarrow 180 t + 360 = 900 t \rightarrow 720 t = 360 \rightarrow t = \frac{360}{720} \rightarrow t = \frac{1}{2} \text{ hour.}\]

**VARIABLE(S):**

\[t = \text{jet’s catch up time.}\]

\[t + 2 = \text{private airplane travel time.}\]

**ANSWER:**

\[t = \frac{1}{2} \text{ hour.}\]
89. Jonathan invests $7,500 @ 10.4% simple interest. How much additional money must he invest at a simple interest rate of 14% so that the total interest earned is 12% of the total investment?

\[
\begin{align*}
\text{Type of Account} & \quad \text{Rate} & \quad \text{Amount Invested} & \quad \text{Interest} \\
10.4\% & \quad 0.104 & \quad 7,500 & \quad 0.104 \times 7,500 \\
14\% & \quad 0.14 & \quad x & \quad 0.14 \times x \\
12\% & \quad 0.12 & \quad 7,500 + x & \quad 0.12 \times (7,500 + x)
\end{align*}
\]

\[
(0.104) \times (7,500) + 0.14 \times x = (0.12) \times (7,500 + x) \quad \Rightarrow \quad 780 + 0.14 \times x = 900 + 0.12 \times x
\]

\[
0.02 \times x = 120 \quad \Rightarrow \quad x = \frac{120}{0.02} \quad \Rightarrow \quad x = 6,000
\]

ANSWER:
Jonathan must invest $6,000 in additional money.

90. Flying with the wind, Rob flew 800 miles between Pittsburgh and Atlanta in 4 hours. The return trip against the wind took 5 hours. Find the rate of the plane in calm air and the rate of the wind.

\[
\begin{align*}
\text{Direction} & \quad \text{Speed} & \quad \text{Time} & \quad \text{Distance} \\
\text{To Atlanta with the wind} & \quad r + w & \quad 4 & \quad 800 \\
\text{From Atlanta against the wind} & \quad r - w & \quad 5 & \quad 800
\end{align*}
\]

\[
(\text{TRAVEL RATE}) \times (\text{TRAVEL TIME}) = \text{TRAVEL DISTANCE}
\]

\[
\begin{align*}
(a) \quad (r + w) \times 4 & = 800 \\
(b) \quad (r - w) \times 5 & = 800
\end{align*}
\]

\[
\begin{align*}
\text{Divide both sides by 4} & \quad \Rightarrow \quad (a) \quad r + w = 200 \\
\text{Divide both sides by 5} & \quad \Rightarrow \quad (b) \quad r - w = 160
\end{align*}
\]

\[
\text{Add both sides} \quad \Rightarrow \quad 2r = 360 \quad \Rightarrow \quad r = 180 \quad \Rightarrow \quad w = 200 - 180 = 20
\]

ANSWER:
180 mph = the rate of plane in calm air.
20 mph = the rate of wind.
91. A member of the City Volunteer Corp. can mow and clean up a large lawn in 9 hours. With two members of the City Volunteer Corp. working, the same job can be done in 6 hours. How long would it take the second member of the team, working alone, to do the job?

**ID – RAG**

<table>
<thead>
<tr>
<th>Member Defined as:</th>
<th>Member’s Work Rate</th>
<th>Time Working Together</th>
<th>Each Member’s Work Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>First member</td>
<td>$\frac{1}{9}$</td>
<td>6</td>
<td>(\left(\frac{1}{9}\right) \times 6)</td>
</tr>
<tr>
<td>Second member</td>
<td>(\frac{1}{t})</td>
<td>6</td>
<td>(\left(\frac{1}{t}\right) \times 6)</td>
</tr>
</tbody>
</table>

\[
\frac{1}{9} \times 6 + \frac{1}{t} \times 6 = 1 \quad \Rightarrow \quad \frac{2}{3} + \frac{6}{t} = 1 \quad \Rightarrow \quad 2t + 18 = 3t \quad \Rightarrow \quad t = 18
\]

\[
\text{LCD} = 3t
\]

**VARIABLE(S):**
\(t = \text{the time needed by the second member to complete the work alone.}\)

**ANSWER:**
18 hrs. = The time needed by the second member to complete the work alone.

92. How many pounds of gourmet candy selling for $1.80 per pound should be mixed with 3 pounds of candy selling for $2.60 a pound to obtain a mixture selling for $2.04 per pound?

**ID – RAG**

<table>
<thead>
<tr>
<th>Type of Candy</th>
<th>Price $ per lb.</th>
<th>Quantity lbs.</th>
<th>Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gourmet candy</td>
<td>1.80</td>
<td>(x)</td>
<td>1.80 (x)</td>
</tr>
<tr>
<td>Other candy</td>
<td>2.60</td>
<td>3</td>
<td>2.60 \times 3</td>
</tr>
<tr>
<td>Mixture</td>
<td>2.04</td>
<td>(x + 3)</td>
<td>2.04 ((x + 3))</td>
</tr>
</tbody>
</table>

\[
\text{COST OF THE CANDY BEFORE MIXING} = \text{COST AFTER MIXING}
\]

\[
1.80 \times x + 2.60 \times 3 = 2.04 \times (x + 3) \quad \Rightarrow \quad 1.80 \times x + 7.80 = 2.04 \times x + 6.12 \quad \Rightarrow \quad 0.24 \times x = 1.68 \quad \Rightarrow \quad x = \frac{1.68}{0.24} \quad \Rightarrow \quad x = 7
\]

**VARIABLE(S):**
\(x = \text{lbs of Gourmet candy}\)
\(x + 3 = \text{lbs of mixture.}\)

**ANSWER:**
7 lbs of Gourmet candy should be mixed.
93. A fenced area is 300 square feet. If the width is 5 feet less than the length, find the length and the width of the fenced area.

\[
\begin{align*}
(1) & \quad A = \text{rectangle's area} \\
& \quad L = \text{rectangle's length} \\
& \quad W = \text{rectangle's width}.
\end{align*}
\]

\[
\begin{align*}
(2) & \quad A = L \cdot W \\
(3) & \quad W = L - 5 \\
(4) & \quad (L - 5) L = 300 \\
(5) & \quad L^2 - 5L = 300 \\
(6) & \quad L^2 - 5L - 300 = 0 \\
(7) & \quad (L - 20)(L + 15) = 0 \\
(8) & \quad L = 20, \quad L = -15 \\
& \quad \text{Reject} \quad -15 \quad \text{(No negative dimension)} \\
(9) & \quad W = L - 5 = 20 - 5 = 15
\end{align*}
\]

ANSWER: The length = 20 ft. The height = 15 ft.

94. At a business meeting at Panera Bread, the bill for two cappuccinos and three house lattes was $14.55. At another table, the bill for one cappuccino and two house lattes was $8.77. How much did each type of beverage cost?

\[
\begin{align*}
(\text{a}) & \quad 2x + 3y = 14.55 \\
(\text{b}) & \quad x + 2(2.99) = 8.77 \\
& \quad x + 5.98 = 8.77 \\
& \quad x = 40 \\
& \quad x = 2.79
\end{align*}
\]

\[
\begin{align*}
-2y & \quad = -17.54 \\
-2y & \quad = -17.54 \\
-2y & \quad = -17.54 \\
-2y & \quad = -17.54 \\
-y & \quad = -2.99 \\
y & \quad = 2.99
\end{align*}
\]

\[
\begin{align*}
\text{VARIABLE(S):} \\
x = \text{cost of one cappuccino.} \\
y = \text{cost of one house latte.}
\end{align*}
\]

\begin{align*}
\text{ANSWER:} \\
\text{One cappuccino costs $2.79.} \\
\text{One house latte costs $2.99.}
\end{align*}