MTH 104: Intermediate Algebra Final Exam Review

Evaluate the function.

1. If \( R(x) = -x^2 - 4x + 8 \), find \( R(2) \)
2. If \( f(x) = x^2 - 3x + 7 \), find \( f(-2) \)
3. If \( g(x) = 2x^3 - 5x \), find \( g(-2) \)

Simplify. Assume all variables are positive.

4. \( \frac{2^{-3}x}{-3x^{-3}y^{-1}} \)
5. \( (3a^{-2})^{-1}(3a^{-1})^3 \)
6. \( (27x^{12}y^{15})^{\frac{2}{3}} \)
7. \( (16y^{12})^{\frac{3}{4}} \)
8. \( \left(\frac{8x^9y^{12}z^{16}}{125}\right)^{\frac{2}{3}} \)
9. \( \frac{\frac{3}{2}a^{-\frac{3}{2}}}{(a^2b^{-1})^\frac{7}{8}} \)
10. \( \sqrt[7]{5a^{15}b^{12}} \)
11. \( \sqrt[12]{-250x^5y^7} \)
12. \( \sqrt{98} \)

Factor completely.

13. \( 15xz^2 + 6x - 5yz^2 - 2y \)
14. \( a^2 - 4ab + 4b^2 \)
15. \( 5x^2 - 14x + 8 \)
16. \( 100y^2 - 49 \)
17. \( 36a^2 + 42ab + 12b^2 \)
18. \( 125a^3 + 8 \)
19. \( 64m^4 - 27mn^3 \)

Perform the indicated operation and simplify. Assume all variables are positive real numbers.

20. \( (x + 3)(x^3 - 2x + 1) \)
21. \( (3a + 2)^2 \)
22. \( (4y - 3)(4y + 3) \)
23. \( \frac{3}{x+5} + \frac{24}{x^2 + 2x - 15} \)
24. \( \frac{4}{x - 2} - \frac{3x + 4}{x - 5} \)
25. \( 3\sqrt{3x^3} + 5x\sqrt{12x} \)
26. \( 5\sqrt{32} - \sqrt{72} \)
27. \( \sqrt{40x} - \sqrt{5}x \)
28. \( \frac{x^2 + 3x - 18}{12 - x - x^2} - \frac{x^2 + 2x - 24}{x^2 + 10x + 24} \)
Divide using long division.

29. \((x^2 - 12x + 32) \div (x - 4)\)  
30. \(\frac{x^3 + 2x^2 - 5x - 6}{x - 2}\)  
31. \(-5 + 12x^2 + 10x) \div (2x + 1)\)

Simplify the fraction.

32. \(\frac{8}{\sqrt{x} - 2}\)  
33. \(\frac{9}{y^2 - 1} - \frac{3}{y} + \frac{1}{1 + y}\)  
34. \(\frac{2 - 4}{y - y^3} + \frac{y}{y - 3}\)

Solve the equations. For the specified variable.

35. \(v = at + v_o\) for \(a\)  
36. \(mv + mp = bv\) for \(v\)  
37. \(s = \frac{a}{1 - r}\) for \(r\)

Solve the equations. Include any complex solutions.

38. \(\sqrt{5x + 1} = 4\)  
39. \(5 + \sqrt{4x + 8} = 11\)  
40. \(\frac{3}{x^2 - 25} + \frac{1}{x + 5} = \frac{8}{x - 5}\)  
41. \(\frac{-6}{a - 6} = 4 - \frac{a}{a - 6}\)  
42. \(x^2 + 196 = 0\)  
43. \(|3x + 2| - 1 = 3\)

Solve each inequality, express the solution using set notation and interval notation, and graph the solution on the real number line.

44. \(-2 < 3x + 7 < 4\)  
45. \(|x - 3| \leq 5\)  
46. \(|5 - x| \geq 2\)  
47. \(x \geq 1\) or \(x > 7\)  
48. \(x < 3\) and \(x \leq 7\)  
49. \(x \geq 4\) or \(x < 7\)  
50. \(x < 3\) and \(x > 10\)  
51. \(x < 0\) or \(x \geq 6\)  
52. \(x \leq 0\) and \(x > -2\)

Solve the quadratic equation by completing the square. Express the solution in set notation.

53. \(x^2 - 6x - 16 = 0\)  
54. \(y^2 + 10y + 22 = 0\)

Solve the quadratic equation by using the quadratic formula. Express the solution in set notation.

55. \(4x^2 = 3 - 8x\)  
56. \(20a - 25 = 4a^2\)  
57. \(x^2 + 2x + 2 = 0\)

Use the discriminant to determine whether the quadratic equation has one real number solution, two real number solutions, or two complex number solutions.

58. \(9x^2 + 30x + 25 = 0\)  
59. \(3t^2 - t + 2 = 0\)
Perform the indicated operation and simplify. Write in \( a + bi \) form.

60. \( 5i(3-2i) \) \hfill 61. \( (3+2i)^2 \) \hfill 62. \( \frac{3+2i}{7-6i} \) \hfill 63. \( \frac{4}{3-7i} \)

Use a calculator to find the value of each of the following. Round your answer to four decimal places.

64. \( \cos 80^\circ \) \hfill 65. \( \tan 14.1^\circ \)

Solve using right triangle ABC with \( \angle C = 90^\circ \).

66. \( c = 3 \text{ cm}, b = 2 \text{ cm}; \)  
   (i) find the exact value of side \( a \)  
   (ii) find the exact values of \( \sin A \), \( \cos A \), \( \tan A \)  
   (iii) find, to the nearest tenth, \( \angle A \) and \( \angle B \).

67. \( A = 28^\circ, b = 5; \) find values of sides \( a \) and \( c \), to the nearest tenth.

68. \( a = 2, b = 5; \) find the exact values of the three trigonometric functions of angle \( A \).

69. \( \text{hypotenuse} = 11, \text{length of one leg} = 7; \) find the three trigonometric functions of the smaller angle (round to 4 decimal places)

70. \( \angle B = 42^\circ, a = 9; \) find all other angles and sides (round to 1 decimal place)

Find the equation of the line that satisfies the information. Write your answer in slope-intercept form.

71. Passes through the point \( (4, -1) \) and has a slope of \( \frac{3}{4} \)

72. Passes through the points \( (-2, 2) \) and \( (1, 3) \)

73. (a) Passes through the point \( (3, 1) \) and is perpendicular to the line \( 2x + y = -3 \)  
   (b) Passes through the point \( (3, 1) \) and is parallel to the line \( 2x + y = -3 \)

Graph the solution to the equations and inequalities. For #76–77, also determine the axis of symmetry, vertex, \( x \)-intercept(s), and \( y \)-intercept.

74. \( x + 2y > 6 \) \hfill 75. \( 3x - 5y < 15 \)

76. \( y = x^2 + 6x + 5 \) \hfill 77. \( y = -x^2 + 4x - 3 \)

Solve the system of equations by graphing.

78. \( \begin{align*} y &= 2x + 1 \\ x + y &= -2 \end{align*} \) \hfill 79. \( \begin{align*} x + 2y &= 6 \\ y &= -\frac{1}{2}x + 3 \)
Solve the system of equations algebraically (use either substitution or addition/elimination method).

80. \[
\begin{align*}
2x + 3y &= 5 \\
x + 2y &= 3
\end{align*}
\]
81. \[
\begin{align*}
2x - 6y &= 5 \\
4x - 12y &= 5
\end{align*}
\]
82. \[
\begin{align*}
x + y - z &= -2 \\
2x - y + z &= -1 \\
x + 2y - 3z &= -7
\end{align*}
\]
83. \[
\begin{align*}
3x - 2y + 4z &= -1 \\
5x - 3y + 5z &= 2 \\
6x - 2y + 3z &= 5
\end{align*}
\]

For the following word problems, identify the variable(s) used, set up an equation(s) and solve algebraically.

84. A 30-foot ladder, leaning against the side of a building, makes a 50° angle with the ground. How far up the building does the top of the ladder reach? Express your answer to the nearest tenth of a foot.

85. A 70 foot rope is attached to the top of one of the vertical poles used to hold up a circus tent. The other end of the rope is anchored to the ground 40 feet from the bottom of the pole. What is the height of the tent and what angle does the rope make with the tent? Express both answers to the nearest tenth (remember about the units).

86. Find three consecutive odd integers such that seven times the sum of the first two integers is three more than nine times the third integer.

87. A chemist must mix 8 L of a 40% acid solution with some 70% acid solution to get a 50% acid solution. How much of the 70% solution should be used?

88. A private airplane leaves an airport and flies due east at 180 mph. Two hours later, a jet leaves the same airport and flies due east at 900 mph. How long will it take for the jet to overtake the private plane?

89. Jonathan invests $7500 at 10.4% simple interest for one year. How much additional money must he invest at a simple interest rate of 14% so that the total interest earned is 12% of the total investment?

90. Flying with the wind, Rob flew 800 miles between Pittsburgh and Atlanta in 4 hours. The return trip against the wind took 5 hours. Find the rate of the plane in calm air and the rate of the wind.

91. A member of the City Volunteer Corp. can mow and clean up a large lawn in 9 hours. With two members of the City Volunteer Corp. working, the same job can be done in 6 hours. How long would it take the second member of the team, working alone, to do the job?

92. How many pounds of gourmet candy selling for $1.80 per pound should be mixed with 3 pounds of candy selling for $2.60 a pound to obtain a mixture selling for $2.04 per pound?

93. A fenced rectangular area is 300 square feet. If the width is 5 feet less than the length, find the length and the width of the fenced area.

94. At a business meeting at Panera Bread, the bill for two cappuccinos and three house lattes was $14.55. At another table, the bill for one cappuccino and two house lattes was $8.77. How much did each type of beverage cost?
ANSWERS TO MTH 104 FINAL EXAM REVIEW

1. \(-4\)  
2. \(17\)  
3. \(-6\)  
4. \(-\frac{x^4y}{24}\)  
5. \(\frac{9}{a}\)  
6. \(9x^8y^{10}\)

7. \(8y^9\)  
8. \(\frac{4x^6y^8z^{10}}{25}\)  
9. \(\frac{1}{a^2b^3}\)  
10. \(5a^7b^6\sqrt{3a}\)  
11. \(-5xy^2 \sqrt{2x^2y}\)

12. \(7i\sqrt{2}\)  
13. \((5z^2 + 2)(3x - y)\)  
14. \((a - 2b)^2\)  
15. \((x - 2)(5x - 4)\)

16. \((10y - 7)(10y + 7)\)  
17. \(6(2a + b)(3a + 2b)\)  
18. \((5a + 2)(25a^2 - 10a + 4)\)

19. \(m(4m - 3n)(16m^2 + 12mn + 9n^2)\)

20. \(x^4 + 3x^3 - 2x^2 - 5x + 3\)

21. \(9a^2 + 12a + 4\)  
22. \(16y^2 - 9\)  
23. \(\frac{3}{x - 3}\)  
24. \(-\frac{3x^2 + 6x - 12}{(x - 2)(x - 5)}\)

25. \(13x\sqrt{3x}\)  
26. \(14\sqrt{2}\)  
27. \(\sqrt[3]{5x}\)

28. \(-\frac{x + 6}{x - 4}\)  
29. \(x - 8\)  
30. \(x^2 + 4x + 3\)  
31. \(6x + 2 - \frac{7}{2x + 1}\)

32. \(\frac{8\sqrt{x} + 16}{x - 4}\)  
33. \(\frac{3 - y}{y}\)  
34. \(\frac{-2y - 6}{4y^3}\)  
35. \(a = \frac{v - v_0}{t}\)

36. \(v = -\frac{mp}{m - b}\) or \(v = \frac{mp}{b - m}\)

37. \(r = -\frac{a - S}{S}\)

38. \(\{3\}\)

39. \(\{7\}\)

40. \(\{-6\}\)

41. No solution; \(\{\}\)

42. \(\{-14i, 14i\}\)

43. \(\left\{-\frac{2}{3}\right\}\)

44. \(\{x \mid -3 < x < -1\} \cup \{-3, -1\}\)

46. \(\{x \mid x \geq 3 \text{ or } x \geq 7\} \cup \{7, \infty\}\)

48. \(\{x \mid x < 3\} \cup \{-\infty, 3\}\)

50. No solution; \(\{\}\)

52. \(\{x \mid -2 < x \leq 0\} \cup \{-2, 0\}\)

53. \(x = -2.8\); \(-2.8\)

45. \(\{x \mid -2 \leq x \leq 8\} \cup [-2, 8]\)

47. \(\{x \mid x \geq 1\} \cup [1, \infty)\)

49. \(\{x \mid x \text{ is a real number}\} \cup (-\infty, \infty)\)

51. \(\{x \mid x < 0 \text{ or } x \geq 6\} \cup (-\infty, 0) \cup [6, \infty)\)

54. \(y = -5 \pm \sqrt{3}; \{-5 - \sqrt{3}, -5 + \sqrt{3}\}\)
55. \[ x = \frac{-2 \pm \sqrt{7}}{2}; \quad \left\{ \frac{-2 - \sqrt{7}}{2}, \frac{-2 + \sqrt{7}}{2} \right\} \]

58. one real solution

60. \( 10 + 15i \)

61. \( 5 + 12i \)

64. \( 0.1736 \)

65. \( 0.2512 \)

67. \( a = 2.7, c = 5.7 \)

68. \( \sin A = \frac{2\sqrt{29}}{29}, \cos A = \frac{5\sqrt{29}}{29}, \tan A = \frac{2}{5} \)

69. \( \sin A = 0.6364, \cos A = 0.7714, \tan A = 0.8250 \)

71. \( y = \frac{3}{4}x - 4 \)

72. \( y = \frac{1}{3}x + \frac{8}{3} \)

74. [Graph of a line]

76. [Graph of a parabola]

77. [Graph of a parabola]

56. \[ \left\{ \frac{5}{2} \right\} \]

57. \(-1 \pm i\)

59. two complex solutions

62. \( \frac{9}{85}, \frac{32}{85} \)

63. \( \frac{6}{29}, \frac{14}{29} \)

66. (i) \( a = \sqrt{5} \)

(ii) \( \sin A = \frac{\sqrt{5}}{3}, \cos A = \frac{2}{3}, \tan A = \frac{\sqrt{5}}{2} \)

(iii) \( \angle A \approx 48.2, \angle B \approx 41.8 \)

70. \( A = 48^\circ, b \approx 8.1, c \approx 12.1 \)

73. (a) \( y = \frac{1}{2}x - \frac{1}{2} \)

(b) \( y = -2x + 7 \)

75. [Graph of a line]

Axis of Sym.: \( x = -3 \)

Vertex: \((-3, -4)\)

x-intercept(s): \((-1, 0)\) and \((-5, 0)\)

y-intercept: \((0, 5)\)
80. (1,1)  
81. Inconsistent system $\Rightarrow$ no solution  
82. (-1, 3, 4)  
83. (1, -4, -3)  

84. The ladder reaches 23.0 feet up the side of the building.  
85. The angle that the rope makes with the tent is 34.8 degrees and the height of the tent is 57.4 feet.  
86. The numbers are 5, 7, and 9.  
87. Four liters of the 70% acid solution should be used.  
88. It will take half an hour.  
89. Jonathan invested $6,000 in additional money.  
90. The rate of plane in calm air is 180 mph and the rate of wind is 20 mph.  
91. The second member would take 18 hours.  
92. Seven pounds of gourmet candy should be mixed.  
93. The length is 20 feet and the width is 15 feet.  
94. Price of a cappuccino is $2.79 & price of a latte is $2.99.