SOLVING ALGEBRAIC EQUATIONS AND FORMULAS

1) Solve by using the square root method
\[(x + 4)^2 = 121\]
\[x + 4 = \pm 11\]
\[x = -4 + 11, -4 - 11\]
\[x = 7, -15\]

5) Solve \[r^{2/5} - 2r^{1/5} - 3 = 0\]

6) Solve \[(x - 2)^{2/3} = x^{1/3}\]

2) Solve by completing the square
a) \[2x^2 + 4x - 1 = 0\]
\[x^2 + 2x - \frac{1}{2} = 0\]
\[x^2 + 2x + 1 = \frac{1}{2} + 1\]
\[(x + 1)^2 = \frac{3}{2}\]
\[x + 1 = \pm \sqrt{\frac{3}{2}}\]
\[x = -1 \pm \sqrt{\frac{3}{2}}\]

7) Solve \[(x^2 + 2x)^{1/3} = x\]

8) Solve \[2x^3 - 7x^2 + 6x = 0\]

9) Solve \[(x - 5)^{2/3} = 16\]

10) Solve \[2m = k - tm\] for \(m\)

11) Solve \[C = \frac{3R + 7}{R}\] for \(R\)

3) Solve
a) \[\sqrt{m + 7} - \sqrt{m - 4} = 3\]
\[\sqrt{m + 7} = 3 + \sqrt{m - 4}\]
\[m + 7 = (3 + \sqrt{m - 4})^2\]
\[m + 7 = 9 + 6\sqrt{m - 4} + m - 4\]
\[6\sqrt{m - 4} = 1\]
\[\sqrt{m - 4} = \frac{1}{6}\]
\[m - 4 = \frac{1}{36}\]
\[m = \frac{1}{36} + 4\]
\[m = \frac{155}{36}\]

b) \[\sqrt{2x + 3} + \sqrt{x - 2} = 2\]

3) Solve
b) \[2x^2 + 6 = 3x\]
\[2x^2 - 3x + 6 = 0\]
\[x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(6)}}{2(2)}\]
\[x = \frac{3 \pm \sqrt{-41}}{4}\]

4) Solve \[3k^4 + 10k^2 - 25 = 0\]

SOLVING INEQUALITIES IN ONE VARIABLE

Solve the given inequality, state the answer algebraically, graph the solution on the real number line and write the answer using interval notation.

12) Solve \[x^2 + 4 \geq 4x\]
\[x^2 - 4x + 4 \geq 0\]
\[(x - 2)^2 \geq 0\]
\[x \leq 2, x \geq 2\]

13) Solve \[3x^2 + 10x < 8\]
\[3x^2 + 10x - 8 < 0\]
\[(3x - 2)(x + 4) < 0\]
\[x < -\frac{1}{3}, x > 2\]

14) Solve \[\frac{x + 3}{x - 1} \leq 0\]
\[x^2 + 2x - 3 \leq 0\]
\[(x + 3)(x - 1) \leq 0\]
\[-3 \leq x \leq 1\]

15) Solve \[\frac{x^2 + 5x}{x - 3} \geq 0\]
\[x^2 + 5x \geq 0\]
\[x(x + 5) \geq 0\]
\[x \leq -5, x \geq 0\]

ALGEBRAIC FUNCTIONS

16) Use \(f(x) = -x^2 + 4x + 2\) and \(g(x) = 3x + 5\) to find:

a) \(f(-3)\)
b) \(f(x - 1)\)
c) \((g - f)(x)\)
d) \((f + g)(3)\)
e) \((fg)(x)\)
f) \((f \circ g)(x)\)

17) Use \(f(x) = \sqrt{x + 13}\) and \(g(x) = x^2 - 4\) to find:

a) \((f \circ g)(x)\)
b) \((f \circ g)(4)\)
c) \((g \circ f)(x)\)
d) \(\left(\frac{f}{g}\right)(x)\)
e) the domain of \(\left(\frac{f}{g}\right)(x)\)
f) the domain of \((f + g)(x)\)
18) Given \( f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 5 \\ x^2 & \text{if } x > 5 \end{cases} \) determine:

   a) \( f(5) \)
   b) \( f(9) \)
   c) \( f(-2) \)

19) State whether \( y \) represents a function of \( x \).

   a) \( x^2 - y = 3 \)
   b) \( y = \sqrt{5 - x} \)
   c) \( (x + 1)^2 = 4y^3 \)
   d) \( x - 3 = y^2 \)
   e) \( y = |x| \)
   f) \( x = |y| \)

20) State the domain using interval notation.

   a) \( y = x^2 + 2x \)
   b) \( y = \frac{2x}{x - 6} \)
   c) \( \frac{5x}{x^2 - 9} \)
   d) \( y = \sqrt{x - 5} \)
   e) \( y = \frac{2x}{\sqrt{x + 3}} \)
   f) \( y = |x| + 3 \)
   g) \( y = \frac{\sqrt{x + 4}}{x - 3} \)
   h) \( y = \log_4 x \)
   i) \( y = 4^x \)

21) Find the inverse \( f^{-1}(x) \) of the given function.

   a) \( f(x) = 12x + 3 \)
   b) \( f(x) = \sqrt{x - 3} \)
   c) \( f(x) = \frac{7}{x - 3} \)
   d) \( f(x) = \frac{x}{x + 2} \)
   e) \( f(x) = \frac{3}{x} + 5 \)

22) Find and state the vertex and axis of symmetry. Write the equation in standard form.

   a) \( y = x^2 - 10x + 13 \)
   b) \( y = 3x^2 + 24x + 56 \)

**SYSTEMS OF EQUATIONS:**

Solve the given system of equations algebraically and graphically.

23) Solve \( \begin{cases} y = x^2 - 2 \\ y = -2x^2 + 6x + 7 \end{cases} \)

24) Solve \( \begin{cases} y + 2x = 1 \\ x^2 + 4x = 6 - 2y \end{cases} \)
GRAPHING

25) Determine if the graph of each equation has symmetry with respect to the x-axis, y-axis and/or origin.
   a) \( y = 4x^2 + 3 \)  
   b) \( x^2 + y^2 = 4 \)  
   c) \( y = x^3 + x \)  
   d) \( y = x + 7 \)  
   e) \( x = y^2 \)

26) Graph the given function. State the domain and range.
   a) \( f(x) = \begin{cases} 
   x^2 - 2 & \text{if } x < 1 \\
   -x + 1 & \text{if } x \geq 1
   \end{cases} \)
   b) \( y = \begin{cases} 
   3x - 2 & \text{if } -4 < x \leq 1 \\
   x^2 - 1 & \text{if } 2 < x \leq 5
   \end{cases} \)
   c) \( y = \left( \frac{1}{2} \right)^{x+1} \)
   d) \( y = \log_3 x \)

27) Given \( f(x) = (2)^x \)
   a) Sketch the graph of \( f(x) \).
   b) State the domain of \( f(x) \).
   c) State the range of \( f(x) \).
   d) State the equation of the horizontal asymptote for \( f(x) \).
   e) State the coordinates of the y-intercept of the graph of \( f(x) \).
   f) Find the equation of the inverse of \( f(x) \).
   g) Graph the inverse function on the same axes as \( f(x) \).

28) Use translations to graph each function. State the horizontal and vertical shifts. Determine the domain and range. Identify the equation of any asymptotes.
   a) \( y = (x - 4)^2 + 3 \)  
   b) \( y = (x + 2)^3 \)  
   c) \( y = \sqrt[3]{x} + 2 \)  
   d) \( y = \ln (x + 1) \)  
   e) \( y = e^x + 3 \)  
   f) \( y = |x + 4| - 2 \)  
   g) \( y = \sqrt{x + 2} - 3 \)  
   h) \( y = 2^{x-3} \)

29) Write the equation in standard form (if necessary) to find the center and radius and graph the circle.
   a) \( x^2 + y^2 = 9 \)  
   b) \( (x - 3)^2 + (y + 5)^2 = 4 \)  
   c) \( x^2 + y^2 + 12x - 2y + 12 = 0 \)
30) Match each equation to its graph:

a) \((x-3)^2 + (y+1)^2 = 16\)  
b) \(f(x) = -2(x+6)^2 - 3\)  
c) \(f(x) = -5x - 4\)

d) \(f(x) = -x^4 + x^3 + 2x^2 + x + 5\)  
e) \(f(x) = \ln x + 4\)  
f) \(f(x) = e^{x^2}\)

g) \(f(x) = \sqrt{x+5} - 3\)  
h) \(f(x) = |x-2| + 4\)  
i) \(f(x) = (x+4)^3\)
31) Determine whether the following graphs show a one-to-one function. For each one-to-one function, sketch its inverse function. Determine the domain and range of each function from its graph.

a) 

\[ y = \sqrt{x} \]

b) 

\[ y = e^{-x} \]

c) 

\[ y = \log_2(x) \]

d) 

\[ y = 3^x \]

LOGARITHMIC / EXPONENTIAL FUNCTIONS

32) Write the exponential equation as an equivalent logarithmic equation

a) \( 3^x = \frac{1}{27} \)  
b) \( e^{4x+2} = 5 \)

33) Write the logarithmic equation as an equivalent exponential equation.

a) \( \log_5 3 = x \)  
b) \( \log x = -4 \)  
c) \( \ln 5 = y \)
34) Simplify each of the following:
   a) \( \log_3 3^5 \)
   b) \( \ln e^{2t} \)
   c) \( \log_{10} 10^4 \)
   d) \( 10^{\log_4} \)
   e) \( e^{\ln 2} \)
   f) \( 5^{\log_4} \)

35) Use the properties of logarithms to expand the expressions.
   a) \( \log (10x) \)
   b) \( \ln (y\sqrt{z}) \)
   c) \( \log \left( \frac{x}{y} \right) \)
   d) \( \log \left( \frac{1}{xy} \right) \)
   e) \( \log_3 \left( \frac{x^2}{x+1} \right) \)
   f) \( \ln \left( \frac{x^2}{3y} \right) \)

36) Use the properties of logarithms to condense the expressions.
   a) \( \log (x+2) + 2 \log x \)
   b) \( \frac{1}{3} \ln a - \ln (c-b) \)
   c) \( 3 \log_5 (x+1) + \log_5 4 \)

37) Solve the following exponential equations.
   a) \( 3^{x-1} = 27 \)
   b) \( 64^x = 8 \)
   c) \( 7^{x^2+x} = 49 \)
   d) \( 2^{x^2} = 512 \)

38) Solve the following exponential equations.
   a) \( 6^t = 12 \)
   b) \( 5^{2r-3} = 17 \)
   c) \( e^{2x-1} + 3 = 7 \)

39) Solve the following logarithmic equations.
   a) \( \log_{64} y = \frac{1}{3} \)
   b) \( \log_2 (y+3) = 5 \)
   c) \( \log x + \log 5 = 1 \)
   d) \( \log 5 - \log x = 2 \)
   e) \( \log(x+21) + \log x = 2 \)
   f) \( \log_4 x + \log_4 (x-30) = 3 \)
   g) \( \log(5x-16) - \log x = \log (x-3) \)
   h) \( \ln (x-5) + \ln (x+1) = \ln \left( x^2 - 9 \right) \)

40) Express each of the following statements as an equation.
   a) \( a \) varies directly as \( b \)
   b) \( p \) varies inversely as \( y \).
   c) \( a \) varies jointly as \( b \) and \( c \).
   d) \( c \) varies directly as \( d \) and inversely as \( g \) and the square of \( f \).
   e) \( J \) varies jointly as \( P \) and the square root of \( T \) and inversely as the cube of \( R \).
41) Solve for the variation constant: \( m \) varies directly as \( z \) and \( p \), and \( m = 10 \) when \( z = 4 \) and \( p = 7 \).

42) \( F \) varies jointly as \( G \) and inversely as the square of \( m \) as stated in the equation \( F = \frac{kG}{m^2} \).

What happens to \( F \) when \( m \) is doubled?

**POLYNOMIAL FUNCTIONS**

43) Given \( P(x) = x^3 + 4x^2 - 5x - 6 \), use synthetic division to find \( P(2) \). Decide if \( x - 2 \) is a factor of \( P(x) \).

44) Given \( P(x) = x^4 - 25x^2 + 144 \), use the Remainder Theorem to determine if \(-3\) is a zero of \( P(x) \).

45) Divide. Use synthetic division where possible. Also state whether or not the 2\(^{nd}\) expression is a factor of the first.
   a) \((x^4 - x^3 - 2x^2 + 10x - 20) \div (x - 2)\)
   b) \((2x^3 + 1) \div (x + 1)\)
   c) \((2x^4 - x^3 + 3x^2 - x + 1) \div (x^2 + 1)\)

46) Find a polynomial function of lowest degree with real coefficients that has the stated zeros. Write the function in standard form.
   a) zeros 3, -2, and 1
   b) zeros 1 and \( 2i \)

47) Find all the zeros of the given function:
   a) \( f(x) = x^3 - 6x^2 + 5x + 12 \) given that 3 is a zero
   b) \( g(x) = 2x^4 + 2x^3 + 16x + 16 \) given that -1 is a zero

48) List all of the possible rational zeros of the given polynomial function. Find all zeros of the function, the \( y \) intercept, and sketch the graph.
   a) \( f(x) = x^3 - 2x^2 - 5x + 6 \)
   b) \( g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4 \)
APPLICATIONS

49) The function \( P(t) = 10,000e^{0.075t} \) estimates the population of a city at time \( t \) years after 1980.
   a) Estimate the population of the city in 1992.
   b) Estimate the year the population doubled its 1980 population.

50) Bacteria cultivated in a laboratory have exponential growth. If a population of 500 bacteria increases to 900 after 1 hour, how many bacteria are present after 7 hours?

51) What amount of money accumulates if Roger puts $3500 in a savings account yielding 6% interest compounded quarterly for 10 years?

52) If Rob puts $5000 in a 5.5% account that is compounded continuously, how much money will he accumulate in 7 years?

53) The intensity, \( I \), of light received at a source varies inversely as the square of the distance, \( d \), from the source. If the light intensity is 20 foot–candles at 15 feet, find the light intensity at 12 feet.

54) The height, \( h \), of a fireworks rocket with an initial velocity of 128 feet per second shot from the top of a 200 foot cliff is a function of time given by the equation \( h(t) = -16t^2 + 128t + 200 \), where \( t \) is the time in seconds.
   a) How long does it take for the rocket to reach its maximum height?
   b) Find the maximum height of the rocket.
   c) How long does it take for the rocket to hit the ground? (Round your answer to the nearest tenth.

55) A farmer plans to build a rectangular pig pen beside his barn, centering it along the broad, eastern side of the barn since pigs love the afternoon shade. He has 88 feet of fencing available. (Note that the side of the pen that is against the barn will not require fencing. Assume that the side opposite the barn is the length of the rectangle.)
   a) Find the dimensions of the rectangle that maximize the enclosed area.
   b) What is the maximum area that can be enclosed by the fence?

56) The volume of a gas in a container at a constant temperature is inversely proportional to the pressure applied. If the volume of the gas is 110 cubic centimeters when the pressure is 6 pounds, what will the volume be when the pressure is 15 pounds?

57) A sample initially contains 850 grams of a radioactive element. Five years later, only 220 grams remain. Assuming the decay is exponential, how much of the radioactive element will be present after eight years? Round your answer to the nearest tenth.